A theoretical explanation as to why the induced rolling moment of an A380 wake on an A320 or A300B2 can be much like that of a B747 wake

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Wakes shortly after rollup
Wakes shortly after rollup

Source: simulation by G. Daeninck (UCL)
We here consider « far field wakes » but « not too far », i.e., before Crow instability:

\[ \tau = 3.0 \]

Case \( N^* = 0 \) and weak atm turbulence

\[ \tau = 4.0 \]

Iso-\( \lambda_2 \) surfaces colored by the axial vorticity

\[ \tau = 5.0 \]

Source: simulation by I. De Visscher (UCL)

Example of velocity field evaluation using WAKE4D results with Crow instability effects (case with significant atmosphere turbulence, thus early development of the instability)

Source: UCL presentation at WN3E specific workshop at TUB
« Not too far field wake » = two-vortex system in equilibrium

- The far field wake of an aircraft, and before the onset of long wave Crow type instabilities, is a « turbulent flow in equilibrium ».
- Such flow consists in a two-vortex system (2VS): two counter-rotating vortices of total circulation $\pm \Gamma_0(t)$ and separated, center to center, by a distance $b_0(t)$.
- Such flow has no memory of the detailed initial conditions.
- Yet, the effective core size is determined by the initial energy (which is itself proportional to the induced drag):
  \[ E(0) = \frac{\Gamma_0^2(0)}{2\pi} \left[ \log \left( \frac{b_0(0)}{r_c(0)} \right) - C \right] \]
- Dimensional analysis then implies that:
  \[ \frac{r_c(t)}{b_0(t)} = f(Re(t)) \quad \text{where} \quad Re(t) = \frac{\Gamma_0(t)}{\nu} \]

As the Reynolds number of aircraft wakes is very high, the function $f(Re)$ no longer varies significantly (i.e., is very flat) with variations of the Reynolds number. Hence, we are left with:
\[ \frac{r_c(t)}{b_0(t)} \approx \text{constant} \]

Examples of 2VS in turbulent equilibrium

Cases with same initial circulation but different core sizes: thus energy and induced drag are different. Source: simulations by I. De Visscher (UCL)
Time evolution of the core radius:

For each case, the curve presents the average of the core radius measurements. Those were measured in each cross-plane, and “sitting” on each local vortex center (port and starboard).

Case 1: \( \frac{r_c}{b_0} = 0.05 \)
Case 2: \( \frac{r_c}{b_0} = 0.1 \)

Rolling moment induced by such 2VS on a follower aircraft

- Induced rolling moment:

\[
M = \frac{1}{2} \rho U_\infty^2 \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty} C_{l,\alpha}(y) c(y) y \, dy
\]

\[
C_M = \frac{M}{\frac{1}{2} \rho U_\infty^2 S b} = \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty} C_{l,\alpha}(y) \frac{c(y)}{c} \frac{y}{b} \, dy
\]

\[
S = b \bar{c}
\]
Further assumptions

- Wing with uniform lift slope: \( C_{l,\alpha}(y) = C_{l,\alpha} \)
- Wing with linear taper: \( \frac{c(y)}{c} = (1 + \beta) - 2\beta \frac{|y|}{b/2} \)
- Remove the non-contribution of the fuselage of diameter \( d_f \)
- One then obtains:

\[
C_M = C_{l,\alpha} \left[ \int_{-b/2}^{-d_f/2} \frac{w_v(y)}{U_\infty} \left( (1 + \beta) + 4\beta \frac{y}{b} \right) \frac{y}{b} dy + \int_{d_f/2}^{b/2} \frac{w_v(y)}{U_\infty} \left( (1 + \beta) - 4\beta \frac{y}{b} \right) \frac{y}{b} dy \right]
\]

“One-scale” models for each vortex of the 2VS

- Definitions:

\[
\begin{align*}
\tilde{u}_\theta(r_c) &= \max(u_\theta(r)) \\
\Rightarrow \tilde{r} &= r/r_c \\
\tilde{\Gamma} &= \frac{\Gamma}{\Gamma_0} \\
\tilde{u}_\theta &= \frac{u_\theta}{\Gamma_0/(2\pi r_c)} \\
\tilde{\omega} &= \frac{\omega}{\Gamma_0/(2\pi r_c^2)}
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{\Gamma}(\tilde{r}) )</th>
<th>( \tilde{u}_\theta(\tilde{r}) )</th>
<th>( \tilde{\omega}(\tilde{r}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian (( \beta = 1.256 ))</td>
<td>( 1 - \exp\left(-\beta \tilde{r}^2\right) )</td>
<td>( \frac{1}{\tilde{r}} \left( 1 - \exp\left(-\beta \tilde{r}^2\right) \right) )</td>
<td>( \beta \exp\left(-\beta \tilde{r}^2\right) )</td>
</tr>
<tr>
<td>Low Order Algebraic</td>
<td>( \frac{\tilde{r}^2}{(\tilde{r}^2+1)} )</td>
<td>( \frac{\tilde{r}}{(\tilde{r}^2+1)} )</td>
<td>( \frac{1}{(\tilde{r}^2+1)^2} )</td>
</tr>
<tr>
<td>High Order Algebraic (( \gamma = 1.781 ))</td>
<td>( \frac{\tilde{r}^2(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2} )</td>
<td>( \frac{\tilde{r}(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2} )</td>
<td>( \frac{2\gamma^2}{(\tilde{r}^2+\gamma)^3} )</td>
</tr>
<tr>
<td>Top Hat ( 0 \leq \tilde{r} &lt; 1 )</td>
<td>( \tilde{r}^2 )</td>
<td>( \frac{\tilde{r}}{1} )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( \tilde{r} \geq 1 )</td>
<td>1</td>
<td>( \frac{1}{\tilde{r}} )</td>
<td>0</td>
</tr>
</tbody>
</table>
“One-scale” models for each vortex of the 2VS

• e.g., Jacquin, Proctor-Winckelmans
• Proctor-Winckelmans model: \( \frac{\Gamma(r)}{\Gamma_0} = 1 - \exp \left( -\frac{\beta_o \left( \frac{r}{b} \right)^2}{1 + \left( \frac{\beta_o}{\beta_i} \left( \frac{r}{b} \right)^{5/4} \right)^{1/p}} \right) \)
  with \( \beta_o = 10, p = 3 \ldots 5, \) and \( \beta_o/\beta_i \) determined by \( r_c/b \)

“Two-scales” models for each vortex of the 2VS

• e.g., Jacquin, Proctor-Winckelmans
• Two-scales models are superior in term of azimuthal velocity and circulation profiles (here calibration from space-developing simulation of WV rollup)

Jackson et al., TP 13629E, 2001
de Bruin and Winckelmans, AW-114-25, Awiator, 2005
Lonfils et al., TR1.1.2-6, Far-Wake, 2008
For simplicity, we here assume that each longitudinally averaged vortex is well represented by a low order algebraic (LOA = B-H) circulation profile:

\[ \Gamma(r, t) = \Gamma_0(t) \frac{r^2}{(r^2 + r_c^2(t))} \]

The induced azimuthal velocity due to one vortex is then:

\[ u_\theta(r, t) = \frac{\Gamma(r, t)}{2\pi r} = \frac{\Gamma_0(t)}{2\pi} \frac{r}{(r^2 + r_c^2(t))} \quad u_\theta(r_c(t)) = \frac{\Gamma_0(t)}{4\pi r_c(t)} \]

The other models could be investigated as well (not done)

Case with wing fully to the left of the left (port) vortex center: contribution of that vortex

Case \( d \geq \frac{b}{2} \):

\[ -\frac{b}{2} \leq y \leq \frac{b}{2} : \quad y + r = d \quad \Rightarrow \quad r = d - y \]

One then obtains:

\[ C_M = \frac{C_{l,\alpha}}{2\pi} \frac{\Gamma_0}{U_\infty b} \left[ \int_{-b/2}^{-d_f/2} \frac{(d - y)}{((d - y)^2 + r_c^2)} \left( (1 + \beta) + 4 \beta \frac{y}{b} \right) \frac{y}{b} dy + \int_{d_f/2}^{b/2} \frac{(d - y)}{((d - y)^2 + r_c^2)} \left( (1 + \beta) - 4 \beta \frac{y}{b} \right) \frac{y}{b} dy \right] \]
Case where right part of wing has passed left (port) vortex center: contribution of that vortex

- Case $0 \leq d < \frac{b}{2}$:
  
  \[
  \begin{align*}
  \frac{d_f}{2} & < y < d \\
  d & = r + y = \frac{d + y}{2} \\
  y & \leq \frac{b}{2} \\
  \end{align*}
  \]

- Can be done as well, as all other cases.

A320 follower (medium)

\[
\begin{align*}
 b & = 34.10 \text{ m} \\
 S & = 122.6 \text{ m}^2 \\
 \bar{c} & = \frac{S}{b} = 3.59 \text{ m} \\
 \beta & \approx 0.53 \\
 d_f & = 3.95 \text{ m}
\end{align*}
\]
A300B2 follower (heavy)

Heavy, yet the smallest!

A300B2-100: MTOW = 137 tons
A300B2-200: MTOW = 142 tons

\[ b = 44.85 \text{ m} \]
\[ S = 260.0 \text{ m}^2 \]
\[ \bar{c} = \frac{S}{b} = 5.79 \text{ m} \]
\[ \beta \approx 0.50 \]
\[ d_f = 5.64 \text{ m} \]

B747-400 leader

\[ b = 64.44 \text{ m} \]
\[ S = 541.2 \text{ m}^2 \]
\[ MTOW = 396 \text{ tons} \]
A380-800 leader

\[
\begin{align*}
b & = 79.8 \text{ m} \\
S & = 845 \text{ m}^2 \\
MTOW & = 560 \text{ tons}
\end{align*}
\]

Relative comparisons

- Compute the following dimensionless quantities:
  - a/c encountering the wake of a/c 1 (B747-400):
    \[
    \frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,1} = G \left( \frac{d}{b}, \frac{d_f}{b}, \frac{r_{c,1}}{b}, \frac{b_{0,1}}{b}, \beta \right) = G_1 \left( \frac{d}{b} \right)
    \]
  - a/c encountering the wake of a/c 2 (A380-800):
    \[
    \frac{U_\infty b}{\Gamma_{0,2}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = G \left( \frac{d}{b}, \frac{d_f}{b}, \frac{r_{c,2}}{b}, \frac{b_{0,2}}{b}, \beta \right) = G_2 \left( \frac{d}{b} \right)
    \]

- Assume a ratio for the circulations:
  \[
  \frac{\Gamma_{0,2}}{\Gamma_{0,1}} = \gamma > 1
  \]

- Hence:
  \[
  \frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = \gamma G_2 \left( \frac{d}{b} \right)
  \]
A320 follower

\[
\frac{\Gamma_{0.2}}{\Gamma_{0.1}} = 1.25, \quad \frac{b_{0.1}}{b_1} = \frac{b_{0.2}}{b_2} = 0.785, \quad \frac{r_{c1}}{b_{0.1}} = \frac{r_{c2}}{b_{0.2}} = 0.04
\]

A320 follower: same but wrt 2VS center

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\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25, \quad \frac{b_{01}}{b_1} = 0.74, \quad \frac{b_{02}}{b_2} = 0.90, \quad \frac{r_{c1}}{b_{0,1}} = \frac{r_{c2}}{b_{0,2}} = 0.04
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\[ d/b \]

\[ G_1, \beta G_2 \]

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Conclusions

- Static analysis of rolling moment induced by « not too far wake » 2VS (i.e., fully developed wake, yet before Crow instabilities)
- Leaders: A380-800 and B747-400, with assumed same $r_c/b_0$ (which can be justified, still subject of ongoing work...)
- Followers: A320 and A300B2
- Even when assuming 25% more circulation for the A380-800 (which is really an upper bound):
  - the rolling moment when the outer part of the wing is within one vortex core (i.e., case $d/b=0.5$) is « much the same ». This is even more so when using more realistic values of $b_0/b$.
  - The rolling moment when the a/c center (and thus the wing center) is inside one vortex core (i.e., case $d/b=0$) is still higher, yet not by 25%. This is even more so when using more realistic values of $b_0/b$.
- Hence: circulation is not the whole story: there is also $r_c$; and $b_0/b$. 