The Probabilistic Two-Phase Wake Vortex Decay and Transport Model – Recent Developments and Applications

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Oberpfaffenhofen

- P2P model design
- comparison with field measurement data and other models
- P3P
- applications / use in wake vortex systems
wake-vortex real-time model – requirements

- fast
- robust
- accurate
- reliable

consider stochastic WV properties
vortex topology
large domain

$1024 \times 1024 \times 1024 \text{ m}^3$

$\epsilon^* = 0.4, N^* = 0, L_t^* = 2.2$

Ingo Hennemann
P2P – Concept

Four pillars of P2P wake vortex model:

- dimensional analysis - characteristic scales: \( b_0, t_0 = 2\pi \frac{b_0^2}{\Omega_0} = \frac{b_0}{w_0} \)

- hydrodynamic basis - laminar decaying potential vortex
  \[
  \frac{\Gamma(r,t)}{\Omega_0} = 1 - \exp\left(-\frac{r^2}{4\nu t}\right)
  \]

- adjustment to LES results of different groups

- calibration with field measurement data \( \Rightarrow \) prediction of pdfs

... well defined circulation: \( \Gamma_{5-15} \)
parameters

P2P accounts for effects of three components of wind, axial- and crosswind shear, turbulence, stable thermal stratification, and ground proximity

input data:  
• a/c: $x_0, y_0, z_0, t_0, V, m, b, \gamma, \psi, \phi$
• meteo: $u(z), v(z), (w(z)), \rho(z_0), q(z), \varepsilon(z), \theta(z)$

output data: $\Gamma(t) \pm \Delta \Gamma(t), y(t) \pm \Delta y(t), z(t) \pm \Delta z(t), [r_c(t), \text{pdfs of } \gamma(t), \psi(t), \phi(t)]$
at a certain flight path position (gate)
model equations — two-phase decay

\[ \Gamma_{5-15}^*(t^*) = A - \exp \left( \frac{-R^*}{v_1^*(t^* - T_1^*)} \right) - \exp \left( \frac{-R^*}{v_2^*(t^* - T_2^*)} \right) \]

- **Diffusion**
- **Rapid decay**

onset of rapid decay:

respective decay rate:

\[ = f(\varepsilon^*, N^*, z^*) \]
**lidar — two-phase decay**

- Slope: -0.10
- Slope: -0.26

\[
\frac{\Gamma_2}{\Gamma(T^*_2,\text{meas})} \quad t^*-T^*_2,\text{meas}
\]
onset of rapid decay $T_2^*$ (OGE)

Sarpkaya, J. Aircraft 37 2000
Holzäpfel, J. Aircraft 43 2006
non-linear relation between circulation and descent speed
⇒ stagnating or even re-bounding vortices in strongly stably stratified environment
crosswind shear gradients

\[ \frac{\partial \Gamma^*}{\partial t^*} = 1.42 \ C(\Gamma^*) \ w^* \ \frac{\partial^2 v^*(z^*)}{\partial z^{*2}} \]

only for transport and only for \( \frac{\partial \Gamma^*}{\partial t^*} < 0 \)

Delisi & Robins AIAA Papers 2006-1075/1076

poor availability of wind shear data limits applicability
ground effect with crosswind

LES Anton Stephan
Image vortices at $z = 1.5 \, b_0$
- GE vortices + images at $z = 0.6 \, b_0$
- after rotation of 180 deg another pair of secondary vortices + images
- $\Gamma_{s,\text{max}}^* = 0.4 \, w^*$

$\mathbf{ground \, effect \, w/o \, crosswind}^1$

$\mathbf{1} \text{Robins, Delisi, & Greene, Journal of Aircraft 38 2001.}$
asymmetric rebound in crosswind conditions

lidar 231 cases / 5210 obs.  
(Holzäpfel & Steen AIAA J. 2007)  
black symbols: LES of Giovannini et al.  
(UPS-IMFT, CENEARO, UCL)

Lee-side vortex rebounds earlier
Luff-side secondary vortex is weaker

adapted secondary vortex parameters
circulation decay in ground proximity

parameterization of P2P:

- onset of rapid decay $0.2 \, t_0$ after reaching $z_{\text{min}}$
- decay rate adjusted to $v_2^* = 0.003$

Holzäpfel & Steen, AIAA J. 45 2007
application examples

minor crosswind
⇒ symmetric rebound
application examples

weak crosswind
⇒ asymmetric rebound

$v^* \approx 1$
application examples
behaviour near obstacles

\[ v^* = 1.5, \quad z_{\text{building}} = 9.7 \text{ m} \Rightarrow \Delta z^* = 0.2 \]

Holzäpfel & Steen, AIAA J. 45 2007
application examples –
exceptional WV rebound due to shear layers

\[ \frac{\partial v^*}{\partial z^*} = 1.8 \]
wake generation below $b_0$ ($z_{0,\text{min}} = 0.1 \ b_0$)
Probabilistic concept – 3 layers

fixed uncertainties:

• variation of decay parameters
  \((v_{2,u}^*, 0.8T_2^*); (v_{2,l}^*, 1.2T_2^*)\)

• uncertainty allowances
  \(\pm b_0, \quad \pm 0.2\Gamma_0\)

dynamic uncertainties:

• uncertainty allowances
  \[
  y_{u(l)}^*, z_{u(l)}^* = y^*, z^* + (-) \int \sqrt{(C_{qq^*})^2 + (C_{ss^*})^2} \, dt^*
  \]

model calibration with measurement data:

• uncertainty allowances (see next slide)
Model Calibration with Measurement Data

prediciton of envelopes with defined probabilities

- calibrate model based on observations
- prediction of pdfs
- adjust degree of probability
- complete statistics within single plot
- optimization
- stochastic model
- different pdfs for OGE & IGE + smooth transition

\[ \hat{Z} = \frac{Z_{\text{meas}} - Z_l}{Z_u - Z_l} \]

Holzäpfel, J. Aircraft 43 2006
probabilistic / stochastic prediction modes
## P2P validation work - survey

<table>
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<tr>
<th>campaign</th>
<th>No. cases</th>
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<td>~ 9,000</td>
<td>X/-</td>
<td>DVM</td>
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<td>CREDOS D2-3, 2009</td>
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<td>OGE/IGE</td>
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Comparison with other WV models

example D2P & DVM (CREDOS)

\( v_c = 4 \text{ m/s} \)
### Prediction Skill

**deterministic scoring results**

A model with perfect (MET) data would be much better.

<table>
<thead>
<tr>
<th>RMS $\Delta \Gamma_{5-15} / \Gamma_0$</th>
<th>RMS $\Delta z / b_0$</th>
<th>RMS $\Delta y / b_0$</th>
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<tbody>
<tr>
<td><strong>best median</strong></td>
<td>0.128</td>
<td>0.118</td>
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<tr>
<td><strong>worst median</strong></td>
<td>0.240</td>
<td>0.452</td>
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<tr>
<td><strong>worst median</strong></td>
<td>86 m²/s</td>
<td>17 m</td>
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WN3E Specific Workshop on Operational WV Models, UCL, Louvain-la-Neuve, 7-8 November 2011

application examples
long-lived vortex rings

circulation decay – vortex topology

- 2-phase decay: shortening of vortex segments in flight direction
- topology caused by mutual velocity induction
- redistribution of $\Gamma_{5-15}$ by vortex stretching and compression and / or collision of pressure waves

$\varepsilon^* = 0.05$, $N^* = 0$, $L_t^* = 0.41$
circulation decay characteristics
continuous to 3-phase decay

- good agreement in stable stratification – large uncertainties in neutral strat.
- neutral - weak stratification: three-phase decay of circulation to vortex line:
  - diffusion phase – rapid decay phase – ring diffusion phase
P3P – long-lived vortex rings, $L_t$
preliminary results (Ingo Hennemann)

\[ \Gamma_{5-15}^{*} = 1 - \left(4.9 \epsilon^{*} + 1.1L_{t}^{*}\right)\exp\left(-\frac{3.4}{\sqrt{t^{*}}}\right) - 0.016\left(N^{*}t^{*}\right)^{2} + C\left(1 + \tanh(t^{*} - T^{*})\right) \]
Wake Vortex Prediction & Monitoring System
WSVBS

strong crosswind

prediction of weather dependent aircraft separations for approach and landing

2004/09/10 19:10
WakeScene – sensitivity analysis, risk assessment

WakeScene-D ⇔ field measurements EDDF-2
lateral WV transport in Lidar plane (∼10,000 departures)

**EDDF-2**

**WakeScene-D**

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**good agreement of *global* vortex properties in lidar measurement domain**

⇒ WakeScene-D supports investigating realistic wake vortex behaviour in domains and height ranges that are far out of reach of measurements
contribution of contrail cirrus to global warming

Annual mean solar optical depth of the sum of contrails computed with CoCiP for the year 2006

Schumann, Graf, Mannstein, AIAA Paper 2011-3376
contribution of contrails to global warming

maximum WV descent distances

\[ \Delta z^* = \frac{1.49}{N^*} \]
for \( N^* \geq 0.8 \)

\[ \Delta z^* = 7.68 \left(1 - 4.07\varepsilon^* + 5.67\varepsilon^{*2}\right)(0.79 - N^*) + 1.88 \]
for \( N^* < 0.8 \), \( \varepsilon^* \leq 0.36 \)

- robust for stable stratification
- substantial uncertainties for neutral stratification


WSVBS, ATC-Wake, SESAR 12.2.2, WakeScene, OWBPA, WEPS, CoCiP, WaVoP coded (partly) by 12 other groups
Conclusions & Outlook

- accounts for relevant effects of aircraft and atmosphere
- prediction of probabilistic uncertainty allowances (pdfs)
- good prediction skill
- comprehensive comparison with field measurement data & other models
- fast (P2P 0.01 s, runtime optimized D2P version OGE 0.001 s on PC)
- consideration of long-lived rings and turbulence length scale
- refinement of ground effect modeling employing measurements at Munich airport and LES of Anton Stephan
Wake Vortex Advisory System “WSVBS” supports weather dependent dynamic a/c separations.

- Approach corridor
- Vortex area
- Safety area (small a/c)
- Safety area (large a/c)

Wake vortex prediction planes

Sodar/rass

Lidar

Air Traffic Control Quarterly, Vol. 17, No. 4, 2009