Aircraft wake vortices: physics and UCL models

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Developed wakes, shortly after rollup
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2VS IGE generated close to the ground and with weak cross wind

\[ h_0 = b_0 \]

\[ V_{\text{wind}}(h_0) = V_0 \]

\[ \tau = 0.0 \]

Simulation by L. Bricteux et al. (UCL)
work with Airbus on further investigations of WV IGE, after UCL work in FAR-Wake
Longitudinally averaged flow

\( \tau = 3.84 \)
Longitudinally averaged flow

Circulation decay (RECAT I model also shown)
2VS OGE and without Crow instabilities

\[
\frac{r_c}{b_0} = 0.05 \quad \quad \quad \frac{r_c}{b_0} = 0.1
\]

Iso-\(\lambda_2\) surfaces colored by the axial vorticity

Cases with same initial circulation but different core sizes: thus energy and induced drag are different. Source: De Visscher et al. (UCL)

Time evolution of the core radius:

For each case, the curve presents the average of the core radius measurements. Those were measured in each cross-plane, and "sitting" on each local vortex center (port and starboard)
2VS with Crow instability

Case $N^* = 0$ and weak turbulence

Iso-$\lambda_2$ surfaces colored by the axial vorticity

Source: De Visscher et al. (UCL), submitted

Case with stratification

Case $N^* = 0.35$ and weak turbulence
Case with stratification

Case $N^*=0.75$ and weak turbulence

$\tau = 2.0$

$\tau = 3.0$

$\tau = 3.5$

Case with stratification

Case $N^*=1.0$ and weak turbulence

$\tau = 1.0$

$\tau = 1.5$

$\tau = 2.0$
Comparison of the longitudinally-averaged circulation evolution

\[ \frac{\bar{\Gamma}_{\text{tot}}}{\Gamma_0} \]

- \( N^* = 0 \)
- \( N^* = 0.35 \)
- \( N^* = 0.75 \)
- \( N^* = 1.0 \)
- \( N^* = 1.4 \)

Comparison of the mean and 95%-envelope circulation evolution

\[ \frac{\Gamma_{\text{tot}}}{\Gamma_0} \]

\[ \frac{\Gamma_{\text{tot,0.6}}}{\Gamma_0} \]

- \( N^* = 0 \)
- \( N^* = 0.35 \)
- \( N^* = 0.75 \)
- \( N^* = 1.0 \)
Improved circulation decay modeling

Stratification effect

Stratification and turbulence effects

\[ N^* \begin{array}{c} \cdot \\ + \\ \times \\ \circ \\ * \\ \end{array} \begin{array}{c} 0 \\ 0.35 \\ 0.75 \\ 1.0 \\ 1.4 \\ \end{array} \]

\[ \epsilon^* = \frac{\epsilon b_0}{V_0^3} = 2.42 \times 10^{-4} \]

\[ \begin{array}{cc} N^* & \epsilon^* \\ * & 0.35 \quad 2.42 \times 10^{-5} \\ + & 0.35 \quad 2.42 \times 10^{-4} \\ \times & 1.0 \quad 2.42 \times 10^{-5} \\ \circ & 1.0 \quad 2.42 \times 10^{-4} \\ \end{array} \]

Improved vortex transport modeling

\[ N^* \begin{array}{c} \cdot \\ + \\ \times \\ \circ \\ * \\ \end{array} \]

\[ \begin{array}{c} 0 \\ 0.35 \\ 0.75 \\ 1.0 \\ 1.4 \\ \end{array} \]

\[ \begin{array}{c} \cdot \\ + \\ \times \\ \circ \\ * \\ \end{array} \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

\[ -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 1 \]

\[ \bar{Q}^* \]

\[ \bar{\Gamma}_{tot} \]

\[ \bar{\Gamma}_{tot} \]
WAKE4D: “3-D space + time” wake vortex prediction platform

- It uses as input:
  - the a/c trajectory
  - the met conditions.
- The computational domain is divided in various “computational gates” crossing the flight path.
- The aircraft crossing one of the gates generates a pair of wake vortices (WV) that are transported (also by the headwind) and also decay.

The Deterministic wake Vortex Model (DVM)

- The DVM forecasts, in real-time, the WV behavior (transport and decay) in one computational gate, using simplified physical models.
- The initial wake is computed using the a/c characteristics (position, mass, TAS, wingspan, lift distribution, and flight angles) when the aircraft crosses the gate.

\[
\begin{align*}
  b_0 &= \frac{s b}{\rho U_{\infty}} \\
  \Gamma_0 &= \frac{Mg}{\rho U_{\infty} b_0} \\
  V_0 &= \frac{\Gamma_0}{2\pi b_0} \\
  t_0 &= \frac{b_0}{V_0}
\end{align*}
\]

- Each primary wake vortex is represented by a vortex particle with a chosen circulation distribution profile:
  - Burnham-Hallock model (= low-order algebraic model),
  - high-order algebraic model, or
  - two-scales Proctor-Winckelmans model
- Alternative: wake vortex sheet model discretized using small vortex particles
“One-scale” models for each vortex of the 2VS

- Definitions:

\[ u_\theta(r_c) = \max(\theta(r)) \quad \Rightarrow \tilde{r} = \frac{r}{r_c} \]

\[ \tilde{\Gamma} = \frac{\Gamma}{\Gamma_0} \quad \tilde{u}_\theta = \frac{\Gamma_0}{\Gamma_0/(2\pi r_c)} \]

\[ \tilde{\omega} = \frac{\omega}{\Gamma_0/(\pi r_c^2)} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(\tilde{\Gamma}(\tilde{r}))</th>
<th>(\tilde{u}_\theta(\tilde{r}))</th>
<th>(\tilde{\omega}(\tilde{r}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian ((\beta = 1.256))</td>
<td>(1 - \exp(-\beta \tilde{r}^2))</td>
<td>(\frac{1}{\tilde{r}} \left(1 - \exp(-\beta \tilde{r}^2)\right))</td>
<td>(\beta \exp(-\beta \tilde{r}^2))</td>
</tr>
<tr>
<td>Low Order Algebraic</td>
<td>(\frac{\tilde{r}^2}{(\tilde{r}^2+1)})</td>
<td>(\frac{\tilde{r}}{(\tilde{r}^2+1)})</td>
<td>(\frac{1}{(\tilde{r}^2+1)^2})</td>
</tr>
<tr>
<td>High Order Algebraic ((\gamma = 1.781))</td>
<td>(\frac{\tilde{r}^2(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2})</td>
<td>(\frac{\tilde{r}(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2})</td>
<td>(\frac{2\gamma^2}{(\tilde{r}^2+\gamma)^3})</td>
</tr>
<tr>
<td>Top Hat (0 \leq \tilde{r} &lt; 1) (\tilde{r} \geq 1)</td>
<td>(\tilde{r}^2) (1)</td>
<td>(\frac{\tilde{r}}{\tilde{r}})</td>
<td>(1)</td>
</tr>
</tbody>
</table>

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Graphs showing the behavior of \(\tilde{\Gamma}\), \(\tilde{u}_\theta\), and \(\tilde{\omega}\) for different models.
“Two-scales” models for each vortex of the 2VS

- e.g., Jacquin, Proctor-Winckelmans
- Proctor-Winckelmans model:
  \[
  \frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left( -\frac{\beta_0 (r/b)^2}{1 + \left( \frac{\beta_0}{\beta_1} (r/b)^{5/4} \right)^{1/p}} \right)
  \]
  \[\text{with } \beta_0 = 10, p = 3 \ldots 5, \text{ and } \beta_0/\beta_1 \text{ determined by } r_c/b\]

- Two-scales models are superior in term of azimuthal velocity and circulation profiles (here calibration from space-developing simulation of WV rollup)

![Graphs showing Comparisons between Different Models](image)

Jackson et al., TP 13629E, 2001
de Bruin and Winckelmans, AW-114-25, Awiator, 2005
Lonfils et al., TR1.1.2-6, Far-Wake, 2008

Circulations: \( \Gamma_0 \) versus \( \Gamma_{5-15} \)

- The WV total circulation \( \Gamma_0 \) is greater than the “Lidar processed” WV circulation \( \Gamma_{5-15} \).
- Model of the initial ratio \( \Gamma_0 / \Gamma_{5-15} \) as a function of the wingspan \( b \):

![Graph showing Initial Ratio vs Wingspan](image)

Source: UCL presentation at WN3E 1st major workshop
Vortex sheet model for Near-Field WVE analysis (also collaboration with TUB and with FAA/DLR)

- Parametric model taking into account:
  - the tip vortex,
  - the outer flap vortex, and possibly
  - the HTP vortex

- The roll-up is calculated using the DVM with several particles

- A routine calculating the near-wake induced velocity field was also developed and used by FAA/DLR

The physical transport models used in the DVM

- **Self-induced velocity:** Biot-Savart law also using image particles (for NGE modeling)

- **Wind convection:** by both the axial and lateral components evaluated at the altitude of WV evolution

- **Wind shear:** the shear of the wind profile is computed and the tilting effect is modeled

- **Stratification effects:** the stratification level, $N^*$, is computed from the temperature profile and the thermally induced rebound is modeled
The physical models used in the DVM

- **Circulation decay**
  - Two phase decay: slow decay phase followed by a rapid decay phase
  - Accumulated "time-to-demise" approach: starting time of the fast decay phase $t_d(EDR, N^*)$
  - The decay rate of both phases also depends on $EDR$ and $N^*$
  - Possibility to also use a TKE-based decay model or the APA decay model using $EDR$

- **Crow instability model**
  - Model of the space developing Crow instability amplitude for predicting the WV deformation
  - Based on the time-to-demise and thus the atmosphere turbulence level

Velocity field evaluation module: case using WAKE4D results with Crow instability effects (case with significant atm turbulence)

Source: UCL presentation at WN3E specific workshop at TUB
Application of DVM to relative comparison of a/c OGE

Variation of weight, wingspan for constant met. conditions: no wind, turbulence \( (\epsilon = 7 \times 10^{-4} \ m^2/\ s^3) \) and no stratification

Example where same loading factor was assumed for both aircraft:

\[
\begin{align*}
    s &= \frac{b_0}{b} = \frac{\pi}{4} = 0.785 \\
    \epsilon_1^* &= \frac{(\epsilon b_0,1)^{1/3}}{V_0,1} = 0.209 \\
    \epsilon_2^* &= \frac{(\epsilon b_0,2)^{1/3}}{V_0,2} = 0.214
\end{align*}
\]

Source: UCL presentation at WN3E 1st major workshop

DVM In Ground Effect (IGE) modeling

- Generate new « secondary » vortex particles close to the ground, to model the ground-generated boundary layer due to no-slip
- Those particles also dynamically separate from the ground
- They interact with the primary vortices and induce the rebound
- There is a redistribution of the secondary particles when the primary vortex has bounced
- An additional IGE decay model, based on a “Particle strength exchange”, enhances the decay of the vortices IGE after rebound.
Comparison between LES and DVM results: case IGE with headwind (from FAR-Wake)

Mean axial vorticity field as computed by a LES

Particles used in the DVM to model WV IGE

The Probabilistic wake Vortex Model (PVM)

- Probabilistic modeling and assessment of wake vortices is operationally required.
- The PVM is an upper software layer, based on a Monte-Carlo approach, using the DVM as a subtool.
- For each PVM run, many DVM runs are performed, with random variations on the impact parameters (each one following its own distribution):
  - met conditions (natural variations and uncertainties)
  - a/c characteristics (uncertainties)
  - some coefficients of the physical models (calibration uncertainties)
PVM outputs

- One then obtains, in a computational gate at each time:
  - a set of vortex positions,
  - their associated circulations.

- Thus: the output distribution is not just a simple image of the input distribution

- A statistical analysis on the result sample can be performed to obtain:
  - PDF, mean, median, variance, percentiles, ...

Bootstrap resampling technique

- Aim: obtain conservative statistical results while limiting the number of DVM runs

- The resampling technique provides an estimate of the variance of the statistics of the Monte-Carlo results

- The “statistics on the statistics” converge fast
  => PVM confidence envelopes are accurate using a moderate number of DVM runs.

- Thus: the PVM approach is computationally efficient, also for real-time systems.
WAKE4D-DVM and WAKE4D-PVM

- The platform can be run deterministically (using the DVM in each gate) or probabilistically (using the PVM in each gate).
- From the 3-D “gate by gate” DVM (resp. PVM) computations, one can rebuild the 3-D wake (resp. envelope of the wake).
- “Control gate”: interpolation in a fixed plane (similar to a LIDAR scanning plane).
- “Control box”: recording of the vortices present in a box as a function of time.

Visualization of the velocity field (velocity norm, also including the wind field part), as predicted in each computational gate using the WAKE4D-DVM

Example of use of WAKE4D : approach to Marseille-Provence

- Aircraft : B747-400
  - MLW : 285,000 kg
  - $b=64.4$ m
  - $s=0.75$ [-]

- Met. conditions :
  - Southern wind profile (log profile)
  - Turbulence (EDR=10^{-4} m^2/s^3 OGE, then log law)
  - No stratification
Aircraft trajectory

- The proper trajectory corresponding to the ac characteristics and the wind is calculated using a WAKE4D “pre-processor” tool.

Computational gate locations

- 144 computational gates with 2.5 s between two gates
- Here we use the WAKE4D-DVM
Rolling moment induced by a 2VS on a follower aircraft

- Induced rolling moment:
  \[ M = \frac{1}{2} \rho U_\infty^2 \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty} C_{l,\alpha}(y) c(y) y \, dy \]

- Contribution of the fuselage of diameter \( d_f \):

\[ C_M = \frac{M}{\frac{1}{2} \rho U_\infty^2 S b} = \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty} C_{l,\alpha}(y) \frac{c(y)}{c} \frac{y}{b} \, dy \]

\[ S = b \bar{c} \]

Further assumptions

- Wing with uniform lift slope:
  \[ C_{l,\alpha}(y) = C_{l,\alpha} \]

- Wing with linear taper:
  \[ \frac{c(y)}{c} = (1 + \beta) - 2 \beta \frac{|y|}{b/2} \]

- Remove the non-contribution of the fuselage of diameter \( d_f \)

- One then obtains:
  \[ C_M = C_{l,\alpha} \left[ \int_{-b/2}^{-d_f/2} \frac{w_v(y)}{U_\infty} \left( 1 + \beta \right) \frac{y}{b} \, dy \quad + \int_{d_f/2}^{b/2} \frac{w_v(y)}{U_\infty} \left( 1 + \beta \right) \frac{y}{b} \, dy \right] \]

\[ \left( 1 + 4 \beta \frac{y}{b} \right) \frac{y}{b} \, dy \]

\[ \left( 1 + \beta \right) - 4 \beta \frac{y}{b} \right) \frac{y}{b} \, dy \]
Model used so far in simplified analysis

• For simplicity, we here assume that each longitudinally averaged vortex is well represented by a low order algebraic (LOA = B-H) circulation profile:

\[ \Gamma(r, t) = \Gamma_0(t) \frac{r^2}{(r^2 + r^2_c(t))} \]

• The induced azimuthal velocity due to one vortex is then:

\[ u_\theta(r, t) = \frac{\Gamma(r, t)}{2\pi r} = \frac{\Gamma_0(t)}{2\pi} \frac{r}{(r^2 + r^2_c(t))} \quad u_\theta(r_c(t)) = \frac{\Gamma_0(t)}{4\pi r_c(t)} \]

• The other models could be investigated as well (not done yet)

Case with wing fully to the left of the left (port) vortex center: contribution of that vortex

• Case \( d \geq \frac{b}{2} \):

\[ -\frac{b}{2} \leq y \leq \frac{b}{2} : \quad y + r = d \quad \Rightarrow \quad r = d - y \]

• One then obtains:

\[
C_M = \frac{C_{l,x}}{2\pi} \frac{\Gamma_0}{U_\infty b} \left[ \int_{-b/2}^{-d_f/2} \frac{(d - y)}{((d - y)^2 + r^2_c)} \left( (1 + \beta) + 4 \beta \frac{y}{b} \right) \frac{y}{b} dy \\
\quad + \int_{d_f/2}^{b/2} \frac{(d - y)}{((d - y)^2 + r^2_c)} \left( (1 + \beta) - 4 \beta \frac{y}{b} \right) \frac{y}{b} dy \right]
\]

• All other cases can be analyzed as well: this was done
**B747-400 leader**

\[
\begin{align*}
    b & = 64.44 \text{ m} \\
    S & = 541.2 \text{ m}^2 \\
    MTOW & = 396 \text{ tons}
\end{align*}
\]

**A380-800 leader**

\[
\begin{align*}
    b & = 79.8 \text{ m} \\
    S & = 845 \text{ m}^2 \\
    MTOW & = 560 \text{ tons}
\end{align*}
\]
Exemple with A320 follower (medium)

Relative comparisons

- Compute the following dimensionless quantities:
  - a/c encountering the wake of a/c 1 (B747-400):
    \[ \frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,1} = G \left( \frac{d}{b}, \frac{d_f}{b}, \frac{r_{c,1}}{b}, \frac{b_{0,1}}{b}, \beta \right) = G_1 \left( \frac{d}{b} \right) \]
  - a/c encountering the wake of a/c 2 (A380-800):
    \[ \frac{U_\infty b}{\Gamma_{0,2}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = G \left( \frac{d}{b}, \frac{d_f}{b}, \frac{r_{c,2}}{b}, \frac{b_{0,2}}{b}, \beta \right) = G_2 \left( \frac{d}{b} \right) \]

- Assume a ratio for the circulations:
  \[ \frac{\Gamma_{0,2}}{\Gamma_{0,1}} = \gamma > 1 \]

- Hence:
  \[ \frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = \gamma G_2 \left( \frac{d}{b} \right) \]
A320 follower, wrt 2VS center: \( d_c = d + b_0/2 \)

\[
\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25, \quad \frac{b_{0,1}}{b_1} = \frac{b_{0,2}}{b_2} = 0.785, \quad \frac{r_{c_1}}{b_{0,1}} = \frac{r_{c_2}}{b_{0,2}} = 0.04
\]
A320 follower, wrt 2VS center

\[ \frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25, \quad \frac{b_{01}}{b_1} = \frac{b_{02}}{b_2} = 0.785, \quad \frac{r_{c1}}{b_{0,1}} = \frac{r_{c2}}{b_{0,2}} = 0.03 \]
Conclusion of simplified static analysis

- Static analysis of rolling moment induced by 2VS (i.e., before Crow instabilities)
- Leaders: A380-800 and B747-400, with same $r_c/b_0$
- Followers: A320 and A300B2
- Even when assuming 25% more circulation for the A380-800 (which is really an upper bound):
  - the rolling moment when the outer part of the wing is within one vortex core (i.e., case $d/b=0.5$) is « much the same ». This is even more so when using more realistic values of $b_0/b$.
  - The rolling moment when the a/c center is inside one vortex core (i.e., case $d/b=0$) is still higher, yet by less than 25%. This is even more so when using more realistic values of $b_0/b$.
- Hence: circulation is not the whole story: there is also $r_c$; and $b_0/b$. 