CALCULATION OF RADAR CROSS SECTION BASED ON SIMULATIONS OF AIRCRAFT WAKE VORTICES


1) ICTEAM, UCL, Louvain-la-Neuve, Belgium,
   carlos.pereira@uclouvain.be, danielle.vanhoenacker@uclouvain.be

(2) Surface Radar Domain, Technical directorate, Thales Air system SA, France,
   david.canal@thalesgroup.com, jean-yves-j.schneider@thalesgroup.com,
   gilles.beauquet@thalesgroup.com, frederic.barbaresco@thalesgroup.com

SESAR project
Outline

- Introduction
- Numerical model
- Simulation procedure
- Example of results
- Conclusion and Perspective
INTRODUCTION
Introduction

• Problematic:
  – Air traffic density expands;
  – Diversity of operational aircraft fleet increases;
  – Airport enlargement is limited;
  – Need to optimise the transportation flow.

• Solution:
  – modulating the safe separation distances according to the weather conditions and airplane types;
  – real-time sensing of wake vortices for the implementation of new air traffic management systems.

• Means:
  – Continuous measurements by Radar and Lidar;
  – Simulation of power backscattered by wake vortices for:
    • better understand the physical mechanics involved;
    • comparison with measurements for calibration.
NUMERICAL MODEL
Overall simulation model

Aircraft detection simulation

= 3D Electromagnetic calculation 

+ Fluid mechanics model

- **Fluid mechanics model:** simulates the movement and evolution of atmosphere parameters (air pressure, air temperature and humidity);

- **Dielectric permittivity** is function of the atmospheric parameters (link between the two models). Semi-empirical formula of Thayer for clear-air:

  $$\Delta \varepsilon_r \approx 2 \cdot 10^{-6} \left[ 0.776 \left( \frac{P_d}{T} - \frac{P_{da}}{T_a} \right) + 0.648 \left( \frac{e}{T} \left( \frac{5827}{T} \right) - \frac{e_a}{T_a} \left( \frac{5827}{T_a} \right) \right) \right]$$

  - $$\Delta \varepsilon_r$$: dielectric permittivity from vortices without ambient air;
  - $$P_d$$: dry air partial pressure [Pa];
  - $$T$$: absolute temperature [K];
  - $$e$$: water vapor partial pressure [Pa];
  - “a” subscript indicates the ambient air.

- **Electromagnetic modeling:** computes the power backscattered to the radar by the wake vortices, evolving in function of dielectric permittivity.
fluid mechanics model

Test case:

• 2D pseudo-spectral numerical method
  – Vortex movement: Incompressible Navier-Stokes equation with Boussinesq approximation;
  – Water vapour: Convection-diffusion equation.

• Output = atmospheric parameters:
  – Water vapor pressure (Pa);
  – Dry air pressure (Pa);
  – Temperature (K).

• Computes the spatial repartition and temporal evolution of the dielectric permittivity $\varepsilon_r$
2D fluid mechanics model

Vortices evolution after roll up

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Electromagnetic model: Radar Cross Section (RCS)

\[ \sigma_r = 10 \log_{10} \left( \frac{k^4}{4\pi} \left| \sum_{1}^{N} \text{Int}_p \right|^2 \right) \]

- \( \sigma_r \): the radar cross section [dB];
- \( k \): wavenumber [m];
- \( N \): number of sub volumes;
- \( \text{Int}_p \): oscillating integral of the \( p^{th} \) sub-volume.

\[ \Rightarrow \text{The overall integral is written as a sum of integrals over sub-domains that fill the entire domain.} \]
Formulas: oscillating integral

\[ \text{Int}_p = \int_{-\hat{r}/2}^{\hat{r}/2} \Delta \varepsilon_{r(y,z)} f(x) \ e^{-i2k\cdot\hat{r}} \ d\hat{r} \]

- \( \hat{r} \) = distance vector \((x, y, z)\) between the receiver and the volume element [m];
- \( \Delta \varepsilon_{r(y,z)} \) = dielectric permittivity from vortices without ambient air (evolving in \( y \) and \( z \) in our case);
- \( f(x) \) = modulation function used to extend the slice in the \( x \) dimension (takes into account the antenna radiation pattern);

\[ \rightarrow 2D \ \Delta \varepsilon_{r(y,z)} * 1D \ f(x) = 2.5D \ \text{approach} \]

- Particularity:
  - At X band frequencies, the wavenumber value is large;
  - Wake vortex detections is performed at several hundred meters.

\[ \rightarrow \text{The calculation consists in solving a highly oscillatory integral (Li method [1]).} \]

SIMULATION PROCEDURE
**Procedure: generality**

**Assumption:**
- The vortices are considered as uniform along the x axis.
Procedure: fence angle consideration

Assumption:
• The vortices are considered as uniform along the x axis.
Vortices descent function and spatial position of radar in respect of the vortices lead to the determination of the vortices age seen by the radar.
Procedure: radar cell extraction

Radar cells extraction following:
- Radar pulse length;
- Radar beam width.
EXAMPLE OF RESULTS
Configuration presented

Three aircraft detection simulation are compared:

- Aircraft 1: \( b_0 = 27 \) m
- Aircraft 2: \( b_0 = 47 \) m
- Aircraft 3: \( b_0 = 63 \) m

For a coherent base of comparison, the following parameters are used:

- Brunt–Vaissala frequency: 0.014 Hz
- Range: 1080 m
- Fence angle: 0 deg and 30 deg
- Elevation angle: 2.5 deg
- Frequency: 9.1 GHz
- Pulse length: 40 m
# Evolution of Radar Cross Section

**Aircraft 1:** width of slice = $6 \times b_0 = 160$ m

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>RCS (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26</td>
<td>-62.18</td>
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<tr>
<td>1.26</td>
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<tr>
<td>1.26</td>
<td>-46.82</td>
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<tr>
<td>1.26</td>
<td>-61.06</td>
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</tbody>
</table>

**Aircraft 2:** width of slice = $6 \times b_0 = 280$ m

<table>
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</thead>
<tbody>
<tr>
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<tr>
<td>1.26</td>
<td>-63.13</td>
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<td>-62.14</td>
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<tr>
<td>1.26</td>
<td>-60.52</td>
</tr>
</tbody>
</table>

**Aircraft 3:** width of slice = $6 \times b_0 = 360$ m

<table>
<thead>
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<th>RCS (dB)</th>
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<tr>
<td>1.26</td>
<td>-58.16</td>
</tr>
</tbody>
</table>
Summary of results: fence = 00 deg

1) Discrimination of radar cells with and without wake vortex possible when reading the RCS evolution.

2) The dynamics is roughly 15 dB.

3) The aircraft 3 is more visible than the aircraft 2 which is more visible than the aircraft 1.
Summary of results: fence = 30 deg

1) Discrimination of radar cells with and without wake vortex possible when reading the RCS evolution.

2) The dynamics is roughly 15 dB.

3) The aircraft 3 is more visible than the aircraft 2 which is more visible than the aircraft 1.

4) Small increases of the RCS values with the fence angle.
CONCLUSION AND PERSPECTIVE
Conclusion

• Electromagnetic calculation is used for the simulation of wake vortex detection.
  – 3D electromagnetic model.
  – Applied on a simplified 2D fluid mechanics model.
    • Physics are extended from 2D to 2.5D, neglecting the variation along the x axis.
    • Fence angles taken into account by stretching the vortices.
  – Evaluation of Radar Cross Section versus time.
  – Use of Li method to solve highly oscillatory integral.

• The aircraft vortices influence are visible on the Radar Cross Section along the propagation path.

• The value of Radar Cross Section fence angle increases slightly with the fence angle.

• Multiple configuration: generation of Look Up Table.
Perspective

- Extend the electromagnetic model to other parameters as Doppler shifts;
- Optimization of electromagnetic computation possible since $\Delta r \gg \lambda$;
- Test the developed algorithm using a Large-Eddy Simulation of wake vortices evolving in realistic atmospheres.
THANK YOU FOR YOUR ATTENTION