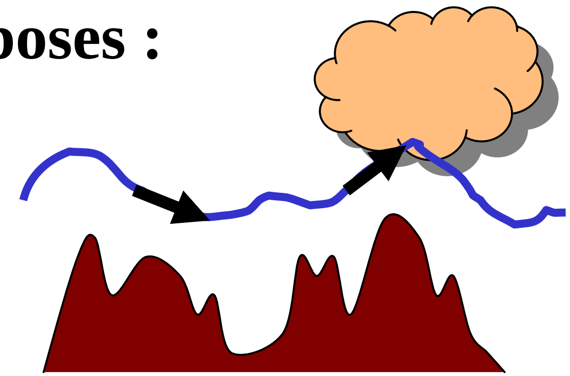
Eddy Dissipation Rate

Towards an improved EDR forecast of the COSMO model for aviation purposes :



- **<u>Model</u> Improvement of direct EDR output by using the concept of scale separation.**
- **<u>Statistical</u> improvement of EDR output by using regression of crucial parameters against EDR measurements.**

Matthias Raschendorfer

DWD, Offenbach

Aim of the DWD-Project:

EDR from separated turbulence model

- **improved by consideration of horizontal shear and related length scales as an additional TKE production**
- **improved by considering the interactions between turbulence and mountain blocking**
- improved by considering the interactions between turbulence and convection

+

Other proper turbulence indicators, like

- Ri-number, vertical gradients of temperature und wind
- Horizontal shear
- large scale surface roughness
- …

connected with remaining **not considered processes** as possible

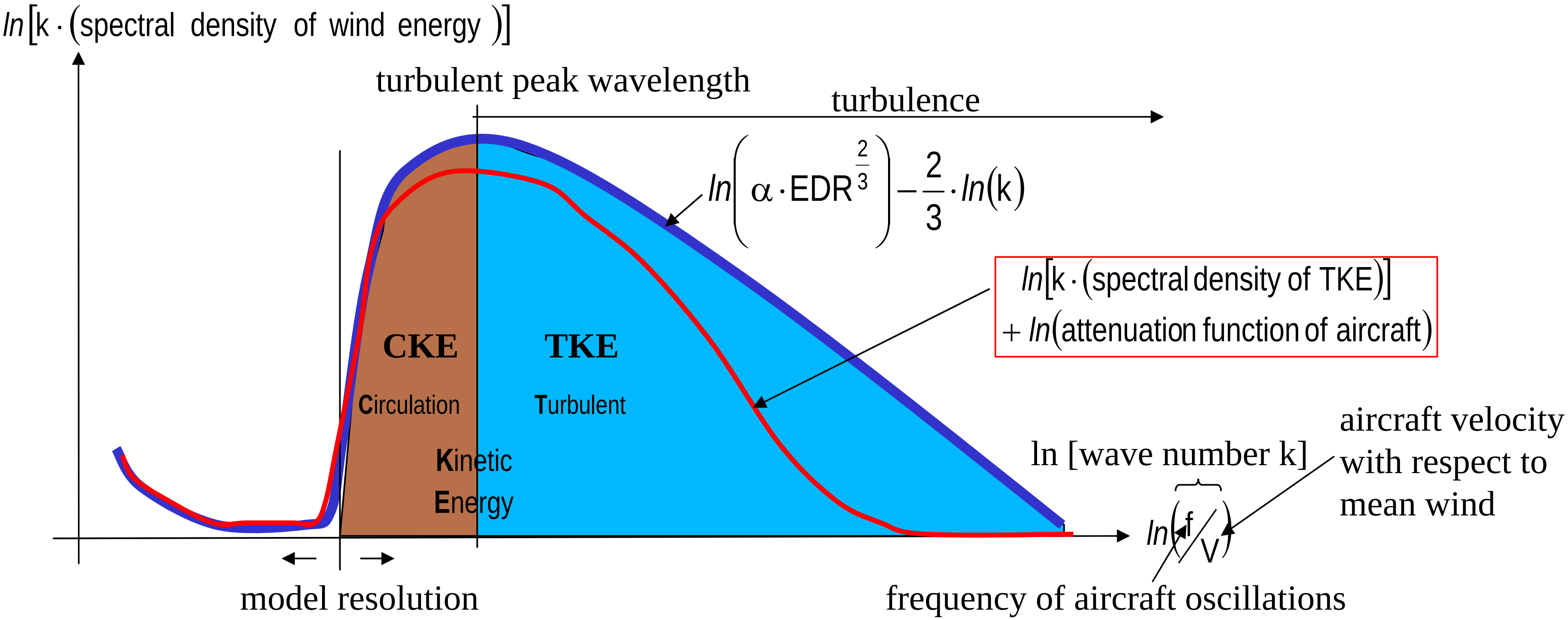
regression model

EDR from aircraft measurements

$$f_{a_1,\ldots,a_n}\left(EDR|_{\mathrm{mod}}, Ri\_nu, Shear, z_0, Div, \ldots\right) \approx EDR|_{mes}$$

↓ **regression parameters**

Aircraft measurements of EDR (from ACARS data base):



spectrum of vertical oscillations

attenuation function
and velocity
of the aircraft

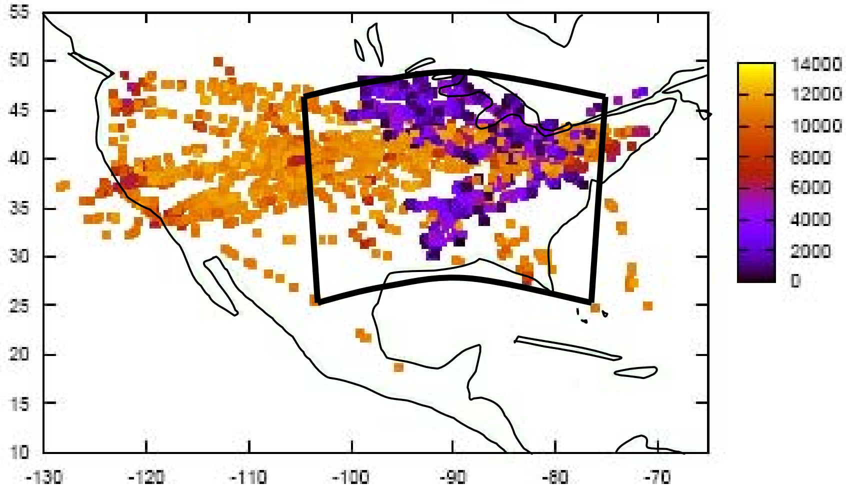
inertial sub range spectrum of atmosphere

EDR by regression of the Kolmogorov spectrum

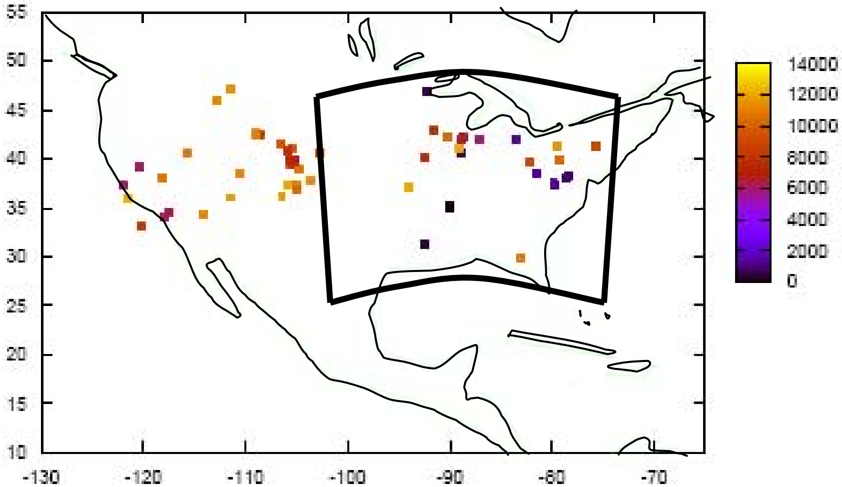
Heights (m) of TURBIDX 1,4,5 events

↕ EDR

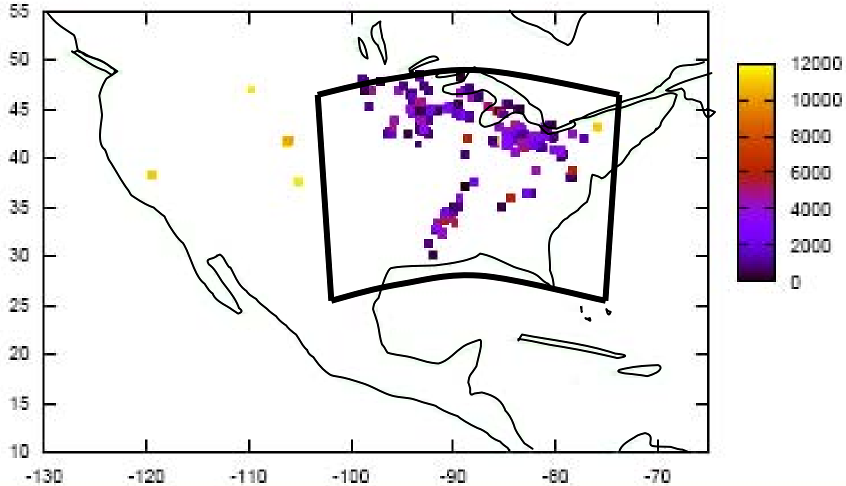
Turbulence index = 1 (light)



Turbulence index = 4 (moderate)



Turbulence index = 5 (severe)



Colours for measurement height in [m]

AMDAR January, 2006

TURBIDX	# Events mo1	# Events wk1
0	1348713	304761
1	9250	2191
2	1070	261
3	188	48
4	66	18
5	336	99
6	27	8
Total	1359649	307386

Turbulence closure on level 2.5 (according to MY):

- For expressing derivatives in space numerically, model equations have to be filtered accordingly, generating 2-nd order moments due to **nonlinearity.**
  They can be described by **2-nd order budget equations**.

- We solve a system of **reduced** **2-nd order equations** (after substituting the turbulent stress tensor by its **traceless** form)
  using special closure assumptions in order to express the unknown statistical moments in those equations

- These assumptions are in accordance only with a **special class** of sub grid scale structures that we call turbulence (mainly arguments of isotropy)

- In particular we neglect all transport terms and time tendency terms

- but we use a **prognostic equation for TKE**

- Closure of **pressure correlation** and **dissipation** terms is done according to **Rotta** and **Kolmogorov** using a **turbulent length scale** ℓ according to **Blackadar**

yielding in particular:

$$\mathrm{EDR} \approx -\frac{q^3}{\alpha^{MM}\ell} \qquad q^2 := 2 \cdot TKE$$

⇨ All reduced second order budgets degenerate to a **system of linear equations**

Single column solution for turbulent flux densities:

1. Using closure assumptions valid for **pure turbulence**:

→ 2-nd order budgets reduce to a 15X15 **linear system** of equations built of all second order moments of the variable set $\{\theta_w, q_w, u, v, w\}$ with only **negligible diabatic source terms**.

2. Using horizontal boundary layer approximation:

- **neglect derivatives** of mean quantities in **horizontal** direction

→ **Flux gradient representation** of the only relevant 2-nd order moments (**vertical** flux densities):

$$\boxed{\overline{\rho\phi'' w''} \approx \bar{\rho}\,\overline{\phi' w'} \approx -\bar{\rho}\underbrace{K^{\phi}\partial_z}_{A^{\phi}}\hat{\phi}}$$

$$K^{\phi} := \ell\, S^{\phi} \cdot q$$ **turbulent diffusion coefficient**

ℓ: turbulent master length scale; S^{ϕ}: stability function; q: turbulent velocity scale $\Big(\underbrace{\frac{1}{\bar{\rho}}\sum_{i=1}^{3}\overline{\rho v_i''^2}}_{2\cdot\mathrm{TKE}}\Big)^{1/2}$

$$\zeta'(\mathbf{s}) := \zeta(\mathbf{s}) - \bar{\zeta}(\mathbf{r}) \qquad \zeta''(\mathbf{s}) := \zeta(\mathbf{s}) - \hat{\zeta}(\mathbf{r})$$

$$\hat{\varsigma} := \frac{\overline{\rho\varsigma}}{\bar{\rho}}$$ **weighted average**

S^M and S^{ϕ} are the **final solution** of the **reduced linear system** and **q** is calculated by a **prognostic equation.**

ℓ is for each horizontal point a pure **function of vertical height** z.

Recent extensions of the COSMO TKE scheme based on scale separation:

- **Separation between turbulence and non turbulent sub grid scale circulations**

↓

- **Additional scale interaction terms in the separated TKE budget**

- **We considered 3 important scale interaction terms with separated:**
  - **Horizontal shear modes** (e.g. at frontal regions)
  - **Wake modes from SSO blocking** (over mountains)
  - **Buoyancy forced thermal circulations** (e.g. due to shallow convection or sub grid scale katabatic flows)

- **Considering of non turbulent sub grid scale circulations in the statistical condensation scheme (including non Gaussian effects)**

Consistent partial solution for turbulence by spectral separation:

Turbulence is that class of **sub grid scale structures** being in **agreement** with **turbulence closure assumptions!**

- Turbulence closure is **only valid** for scales **not larger than**
  - the **smallest** peak wave length L_p of inertial sub range spectra from samples in **any direction** ($L_p \propto \ell < L_p$)
  - the **largest** (horizontal) dimension D_g of the control volume

→ ▪ **Spectral separation** by
  - considering **budgets** with respect to the **separation scale** $L = \min\{L_p, D_g\}$
  - **averaging** these budgets along the **whole control volume** (**double averaging**)

→ **generalized turbulent budgets** including **additional scale interaction terms**

Consistent partial solution for turbulence by spectral separation:

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  - the **largest** (horizontal) dimension D_g of the control volume

→ - **Spectral separation** by
  - considering **budgets** with respect to the **separation scale** $L = \min\{L_p, D_g\}$

→ Filter is an **moving volume average** with infinitesimal vertical extension and horizontal dimension L **.**

  - **averaging** these budgets along the **whole control volume** (**double averaging**)

→ **generalized** turbulent budgets including **additional scale interaction terms**

Additional circulation terms in the turbulent 2-nd order budgets:

average of the **non linear** turbulent **shear terms**

$$D_t\left(\overline{\overline{\rho\phi''\psi''}\Big|_L}\right) = \cdots \left[-\overline{\overline{\rho\phi''\mathbf{v}''}\Big|_L \cdot \left(\overline{\nabla}\hat{\psi}\right)\Big|_L} \quad + \quad -\overline{\overline{\rho\psi''\mathbf{v}''}\Big|_L \cdot \left(\overline{\nabla}\hat{\phi}\right)\Big|_L} \right] + \cdots$$

$$\underbrace{-\overline{\overline{\rho\phi''\mathbf{v}''}\Big|_L} \cdot \overline{\nabla}\hat{\psi}}_{\text{turbulent shear term}} \quad \underbrace{-\overline{\overline{\rho\phi''\mathbf{v}''}\Big|_L{}' \cdot \left(\overline{\nabla}\hat{\psi}\right)\Big|_L{}'} - \overline{\overline{\rho\psi''\mathbf{v}''}\Big|_L{}' \cdot \left(\overline{\nabla}\hat{\phi}\right)\Big|_L{}'}}_{S_C^{\phi\psi}} \quad \underbrace{-\overline{\overline{\rho\psi''\mathbf{v}''}\Big|_L} \cdot \overline{\nabla}\hat{\phi}}_{\text{turbulent shear term}}$$

turbulent
shear term

$S_C^{\phi\psi}$
circulation
shear term

turbulent
shear term

$\Big|_L$: with respect to the separation scale L

Physical meaning of the circulation term:

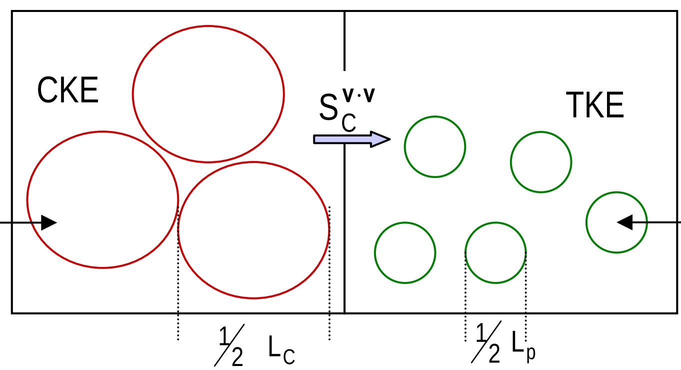
- **Budgets for the circulation structures**:

$$D_t\left(\overline{\rho\phi''\psi''} - \overline{\overline{\rho\phi''\psi''}\big|_L}\right) = \cdots + \underbrace{\left[\overline{\overline{\rho\phi''\tilde{\mathbf{v}}''}\big|_L{}' \cdot \left(\overline{\nabla}\hat{\psi}\right)\big|_L{}'} + \overline{\overline{\rho\psi''\tilde{\mathbf{v}}''}\big|_L{}' \cdot \left(\overline{\nabla}\hat{\phi}\right)\big|_L{}'}\right]}_{-S_C^{\phi\psi}} + \cdots + Q^{\phi\psi} \approx 0$$

Circulation term is the **scale interaction term** shifting Co-Variance (e.g. **S**ub grid scale **K**inetic **E**nergy) form the circulation part of the spectrum (**CKE**) to the turbulent part (**TKE**) **by virtue of shear** generated by the circulation flow patterns.

production terms dependent on:

specific length scales L_C and **specific velocity scales** (= $\sqrt{CKE}$)



production terms depend on:

the **turbulent length scale** L_p and the **turbulent velocity scale** (= $\sqrt{TKE}$)

and other

circulation-scale | **turbulence**-scale

statistical moments

and other

We need to consider additional length scales besides the turbulent length scale!

Separated semi parameterized TKE equation (neglecting laminar shear and transport):

$\big|_L$: with respect to the separation scale L

Γ : correction factor in case of sloped model layers

buoyant part of $-\overline{\mathbf{v}'' \cdot \nabla p|_L}$

mean (horizontal) **shear** production of circulations, **buoyant** and **wake** part of $-\overline{\hat{\mathbf{v}}|_L{}'' \cdot \left(\overline{\nabla} \bar{p}\right)|_L}$

$-\mu \sum_{i=1}^{3} \overline{|\nabla v_i|^2}$

expressed by **turbulent** **flux gradient** solution

to be parameterized by a **non turbulent** approach

according Kolmogorov

$$\partial_t\left(\frac{1}{2}\bar{\rho}\cdot\overline{q|_L^{\,2}}\right) = \frac{1}{2}\overline{\nabla}\cdot\left(\overline{\bar{\rho}\, q|_L^2\, \hat{\tilde{\mathbf{v}}}} + \sum_{i=1}^{3}\overline{\left(\rho v_i''^2 \tilde{\mathbf{v}}''\right)|_L}\right) + \frac{g}{\hat{\theta}_v}\overline{\rho\theta_v'' w''|_L} + \left[-\sum_{i=1}^{3}\overline{\rho v_i'' \tilde{\mathbf{v}}''|_L}\cdot\overline{\nabla}\hat{v}_i\right] + \left[-\sum_{i=1}^{3}\overline{\rho v_i'' \tilde{\mathbf{v}}''|_L{}' \cdot \left(\overline{\nabla}\hat{v}_i\right)|_L{}'}\right] + \left[-\bar{\rho}\frac{\overline{q|_L}^{\,3}}{\alpha^{MM}\ell}\right]$$

Buoyancy term: $=: -\bar{\rho}\,\overline{q|_L}\,\ell S^H F^H$

Shear term by the mean flow: $=: \bar{\rho}\,\Gamma\,\overline{q|_L}\,\ell S^M F^M$

Circulation term: $=: \bar{\rho}\,\Gamma\,\overline{q|_L}\,\ell S^M F_C^M$

time tendency

transport (advection + diffusion)

buoyancy production

labil: > 0

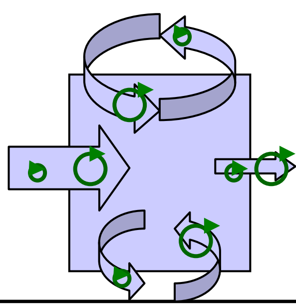
neutral: $= 0$

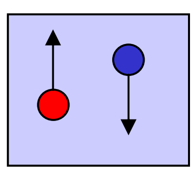
stabil: < 0

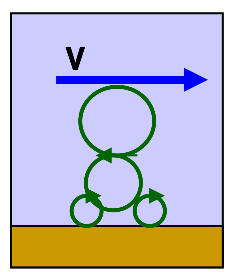
shear production by the mean flow ≥ 0

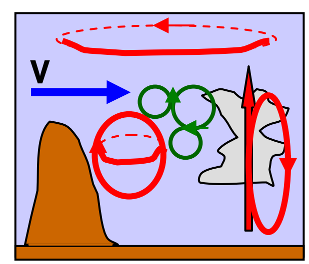
shear production by sub grid scale circulations ≥ 0

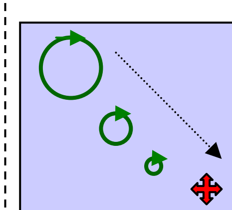
eddy-dissipation rate (**EDR**) < 0



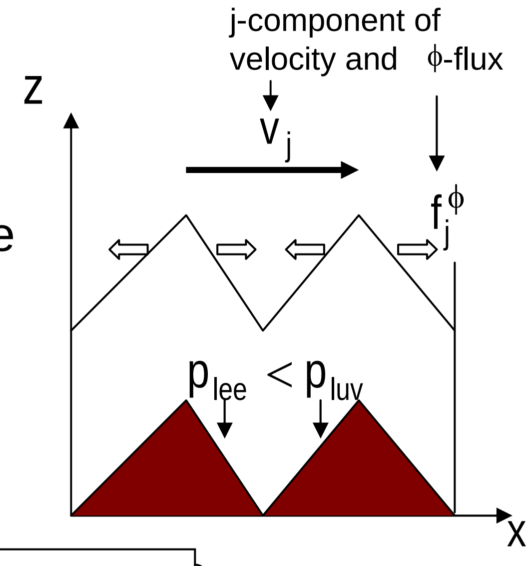








Volume averaging and spatial gradient:

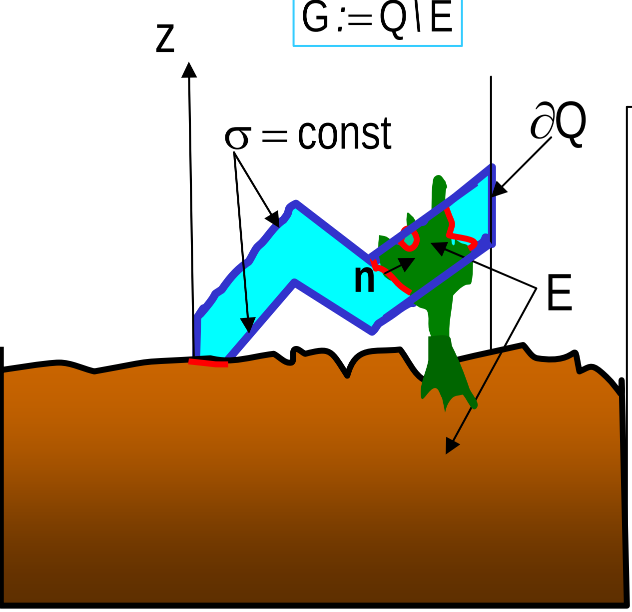


transformed grid scale gradient | sub grid scale gradient correction

$$\overline{\partial_j \zeta} = \frac{1}{|G(\mathbf{r})|}\int_{\mathbf{s}\in\partial G(\mathbf{r})} \zeta(\mathbf{s})\, n_j\, d^2 s = \overline{\partial}_j\, \overline{\zeta} + \overline{\partial'_j \zeta'}$$

$$\overline{\partial}_j\, \overline{\zeta} = \partial_j \overline{\zeta}|_\sigma - \partial_z \overline{\zeta}\, \overline{\partial_j z}|_\sigma$$

$G := Q \setminus E$



Sub grid scale gradient correction $\overline{\partial'_j \zeta'}$:

$$-\partial_z \overline{\zeta (\partial_j z|_\sigma)'}$$

$$-\overline{\zeta (\partial_j z|_\sigma)'}\; \partial_z \ln(q^a)$$

$$+\; \overline{\zeta}\; \overline{\partial}_j \ln(q^a)$$

$$+\; \frac{1}{|G|}\int_{\mathbf{s}\in B} \zeta(\mathbf{s})\, n_j\, d^2 s$$

transformation correction (first two terms)

volume terms (first three terms)

intersection correction (last three terms), with $q_a = \frac{|G|}{|Q|}$

$$\frac{1}{|G|}\int_{\mathbf{s}\in B} \zeta'(\mathbf{s})\, n_j\, d^2 s$$

$B := \partial E \cap \overset{\circ}{Q}$

TKE-production by separated wake modes due to SSO:

- **SSO-term in filtered momentum budget:**

$$\partial_t\left(\bar{\rho}\,\hat{v}_i\right) = -\nabla\cdot\left[\bar{\rho}\left(\hat{v}_i\hat{\mathbf{v}} + \overline{\rho v_i''\mathbf{v}''} - \upsilon\nabla\hat{v}_i\right)\right] - \bar{\rho}g - \partial_i\bar{p} \underbrace{-\overline{\partial_i' p'}}$$

blocking term $Q_{SSO}^{v_i}$

currently Lott und Miller (1997)

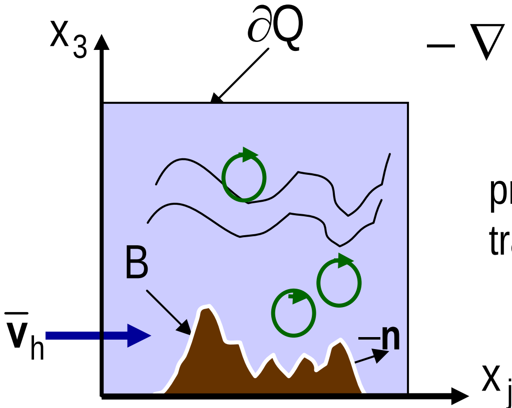
↕

- **Pressure term in kinetic energy budget:**

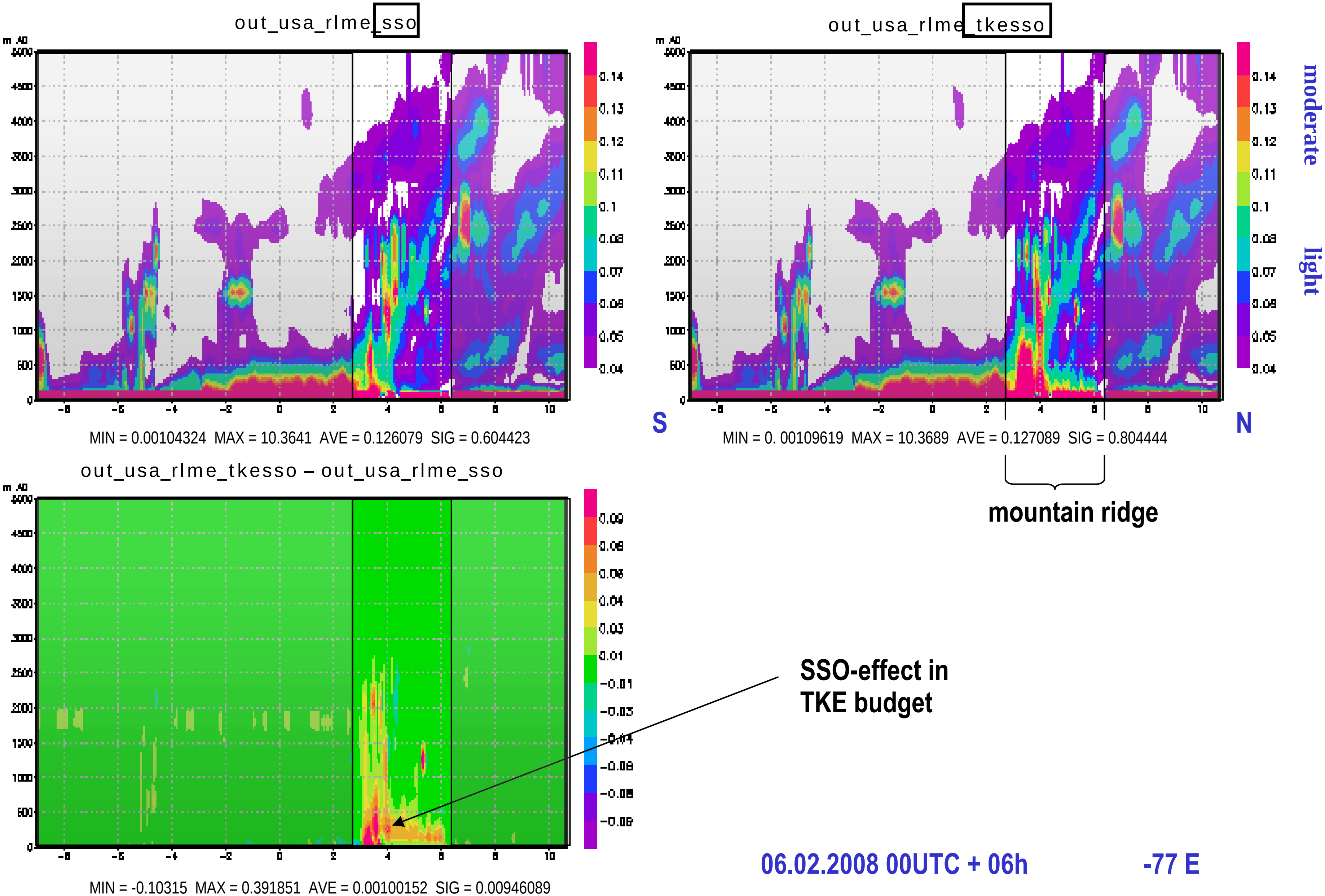
from inner energy: $+\bar{p}\,\bar{\nabla}\cdot\hat{\mathbf{v}}$, $+\overline{p'\nabla\cdot\mathbf{v}}$, $+\bar{p}\,\overline{\nabla'\cdot\mathbf{v}'}$

$$-\overline{\mathbf{v}\cdot\nabla p} = -\bar{\nabla}\cdot\left(\bar{p}\hat{\mathbf{v}}\right) - \bar{\nabla}\cdot\overline{\mathbf{v}p'} - \overline{\nabla'\cdot\left(\mathbf{v}p\right)'} - \overline{\mathbf{v}''}\cdot\bar{\nabla}\bar{p} + \bar{p}\,\bar{\nabla}\cdot\hat{\mathbf{v}} + \overline{p'\nabla\cdot\mathbf{v}} + \bar{p}\,\overline{\nabla'\cdot\mathbf{v}'} - \hat{\mathbf{v}}\cdot\overline{\nabla'p'} + \hat{\mathbf{v}}\cdot\overline{\nabla'p'}$$

pressure transport	buoyancy production	expansion production	wake source
$-\bar{\nabla}\cdot\left(\bar{p}\hat{\mathbf{v}}\right)$		$+\bar{p}\,\bar{\nabla}\cdot\hat{\mathbf{v}}$	$-\hat{\mathbf{v}}\cdot\overline{\nabla'p'}$ (sources of **mean** kinetic energy MKE)
$-\bar{\nabla}\cdot\overline{\mathbf{v}p'}$	$-\overline{\mathbf{v}''}\cdot\bar{\nabla}\bar{p}$	$+\overline{p'\nabla\cdot\mathbf{v}}$	$+\hat{\mathbf{v}}\cdot\overline{\nabla'p'}$ (sources of **sub** grid scale kinetic energy SKE)
$-\overline{\nabla'\cdot\left(\mathbf{v}p\right)'}$		$+\bar{p}\,\overline{\nabla'\cdot\mathbf{v}'}$	



eddy dissipation parameter [m^(2/3)/s] = $(\text{dissipation})^{1/3}$



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TKE-production by separated horizontal shear modes:

- **Separated horizontal shear production term:**

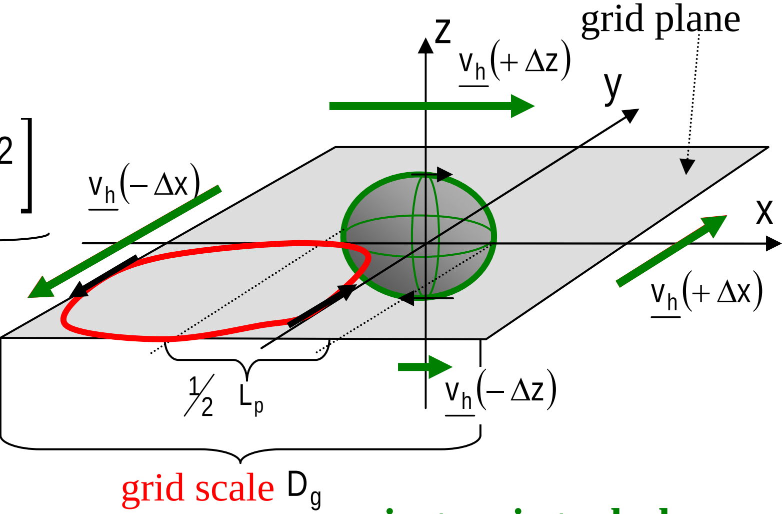
$$S^{\mathbf{v}\cdot\mathbf{v}}_{C\_SHS} := q_H \cdot \underbrace{\beta_H D_g}_{} \cdot \underbrace{\left[\left(\partial_1 \bar{v}_2 + \partial_2 \bar{v}_1\right)^2 + 2\left(\partial_1 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_2\right)^2\right]}_{=:\, F_H^M}$$

separated horizontal shear

effective mixing length of diffusion by horizontal shear eddies

velocity scale of the separated horizontal shear mode

$\beta_H < 1$ **scaling parameter**



isotropic turbulence

horizontal shear eddy

- **Equilibrium of production and scale transfer towards turbulence:**

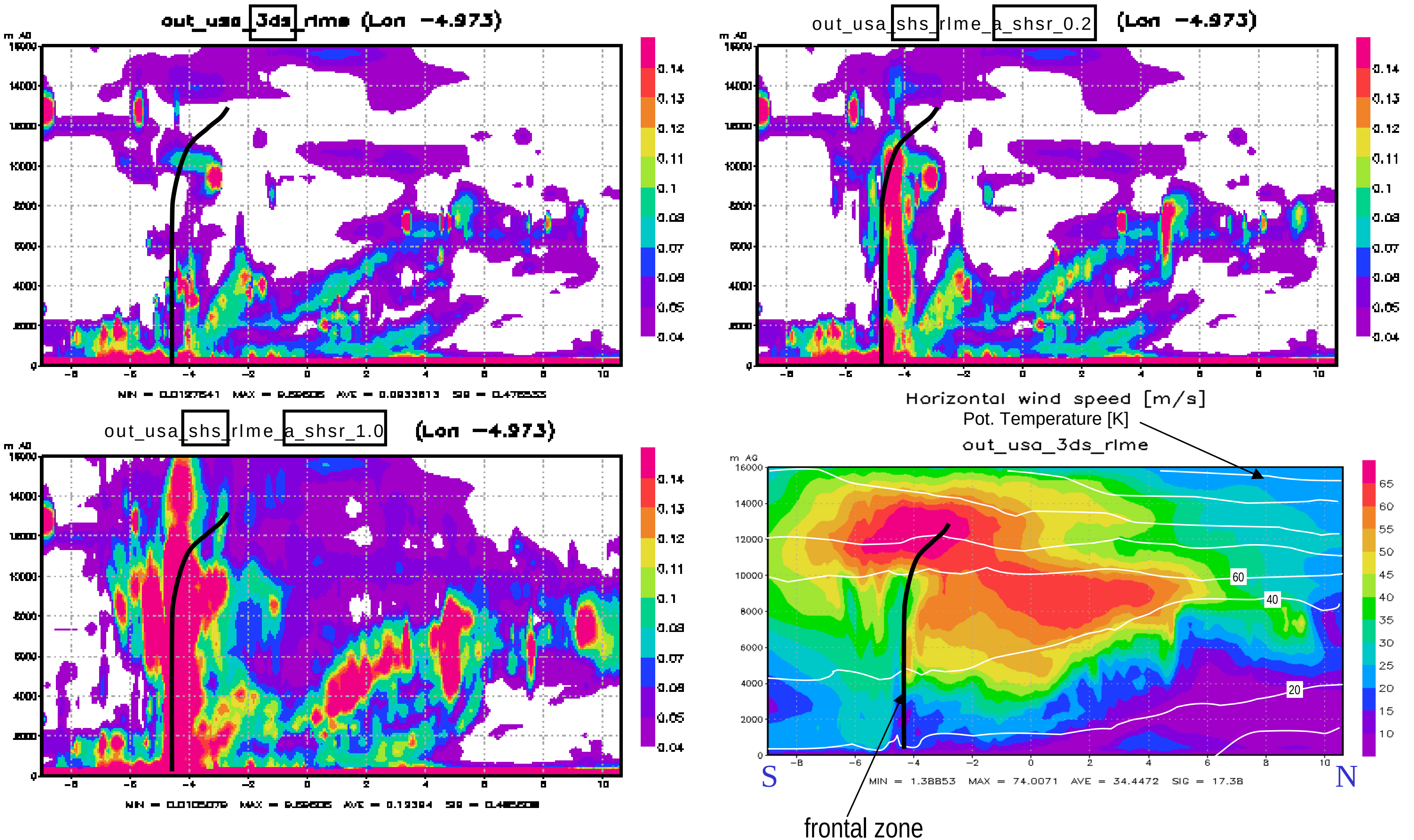
$$q_H \cdot \beta_H D_g \cdot F_H^M = \frac{q_H^3}{\alpha_H D_g}$$

$\alpha_H < 1$ **scaling parameter**

$$\Rightarrow\; S^{\mathbf{v}\cdot\mathbf{v}}_{C\_SHS} = q_H \beta_H D_g \cdot F_H^M = \underbrace{\alpha_H^{\frac{1}{2}} \beta_H^{\frac{3}{2}}}_{=:\, \alpha_S^2} D_g^2 \cdot F_H^{M\,\frac{3}{2}}$$ **additional TKE source term**

$=: \alpha_S^2$ ……….**effective scaling parameter**

eddy dissipation parameter [m^(2/3)/s] = (dissipation)$^{1/3}$



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TKE-Production by thermal circulations:

- **Circulation scale 2-nd order budgets with proper approximations valid for thermals**:

→ **circulation scale temperature variance ~ circulation scale buoyant heat flux ≈ circulation term**

separated thermals; vertical velocity scale of circulation; virtual potential temperature of ascending air

$$\overline{\overline{\rho}|_L \hat{\theta}_V|_L''^2} \propto S^{v\cdot v}_{C\_STH} \approx \frac{g}{\hat{\theta}_v} \cdot \overline{\overline{\rho}|_L \hat{w}|_L'' \hat{\theta}_V|_L''} \approx \frac{g}{\hat{\theta}_v} \cdot \overline{\rho}\, w_C(\sigma) \cdot \left(\theta_v^+ - \theta_v^-\right) \geq 0$$

virtual potential temperature of descending air

$$w_C(\sigma) = \gamma_C \cdot \left(g \frac{\theta_v^+ - \theta_v^-}{\hat{\theta}_v} H_C \right)^{1/2} exp\left(-\frac{\sigma - \sigma_b}{H_C - \sigma_b} \right)$$

scaling factor; circulation height: e.g. BL-height; bottom level

- **Simplified max flux approach for the circulations:** (using an friction-buoyancy equilibrium for relative updraft velocity $w^+ - \hat{w}$)

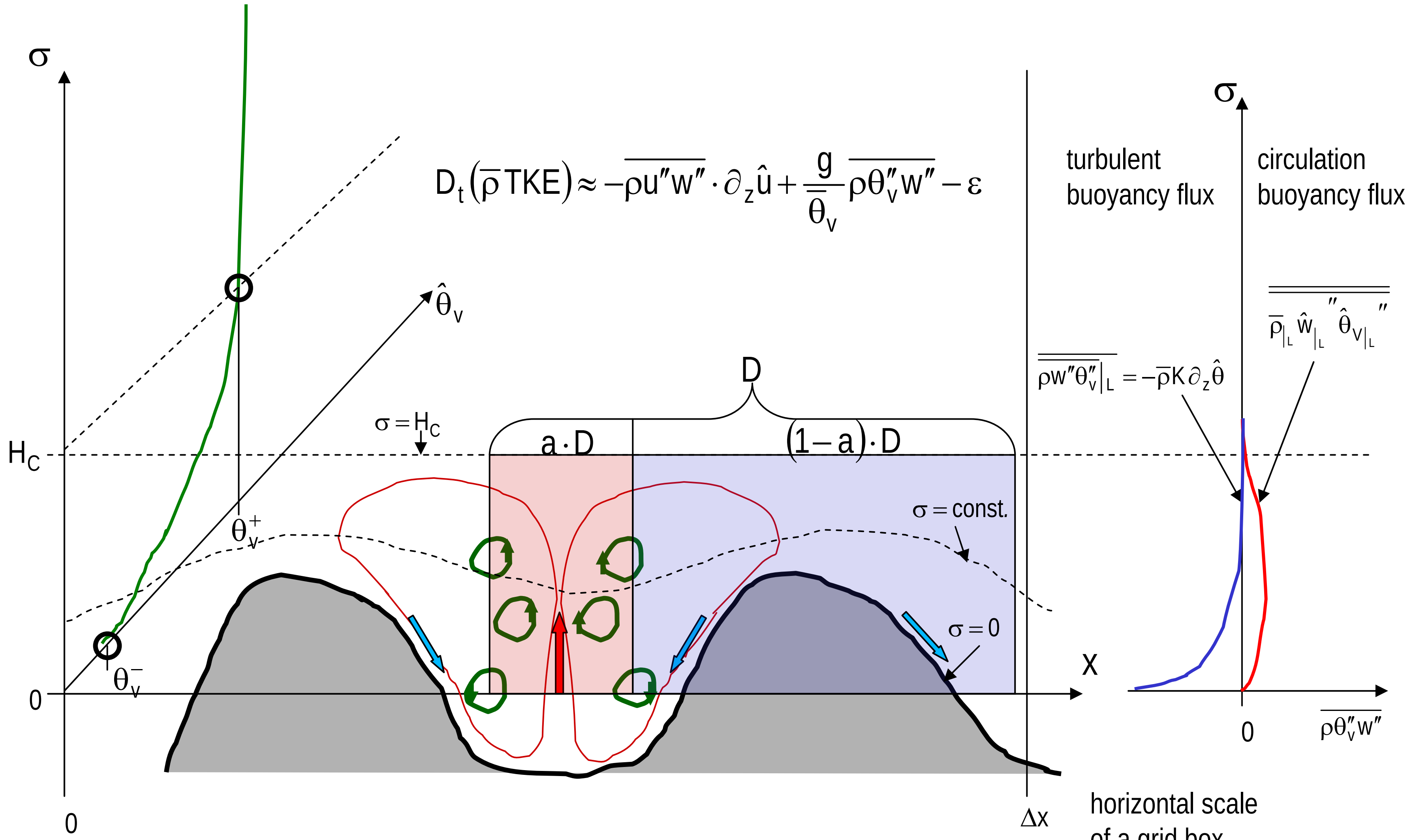
horizontal scale of an individual updraft

$$w_C \approx \underbrace{\frac{\hat{\theta}_v - \theta_v^-}{\theta_v^+ - \theta_v^-}}_{\text{horizontal updraft fraction } a} \cdot \underbrace{\frac{L^+ g}{q} \cdot \frac{\hat{\theta}_v^+ - \hat{\theta}_v}{\hat{\theta}_v}}_{w^+ - \hat{w}}$$

$q = \sqrt{2TKE}$ turbulent velocity scale

$$\hat{\theta}_v^+, \hat{\theta}_v^- \in \left\{ \hat{\theta}_v(0), \hat{\theta}_v(H_C) \right\}$$

Effect of the thermal circulation term for stabile stratification:



- Even for **vanishing mean wind** and **negative turbulent buoyancy** there remains a **positive definite source term**

→ TKE will **not** vanish → Solution even for **strong stability**

Conclusion:

- **3D-shear terms** have got a significant effect **only**, when formulated as a **scale interaction term** producing TKE by shear of a **separated horizontal shear mode** with its **own length scale**

- **Wake production** of TKE by **blocking** can be formulated as a **scale interaction term** as well and can be described by **scalar multiplication** of the **horizontal wind vector** with its **SS0-tendencies** yielding some effect above mountainous terrain.

Prospect:

- Derivation and implementation a similar **scale interaction term** from the **convection** scheme as well.
- Next we plan to introduce a **vertically resolved roughness layer** including a more sophisticated **interaction between turbulence and topographic blocking**.
- Final steps in setting up a developed **program package for calculation derived turbulence indices** form the model output and for relating them to aircraft measurements.
- Using the related model and measurement data for **verification of the implemented model extensions** and as an input of the regression procedure.

Thanks for your attention!

Motivation:

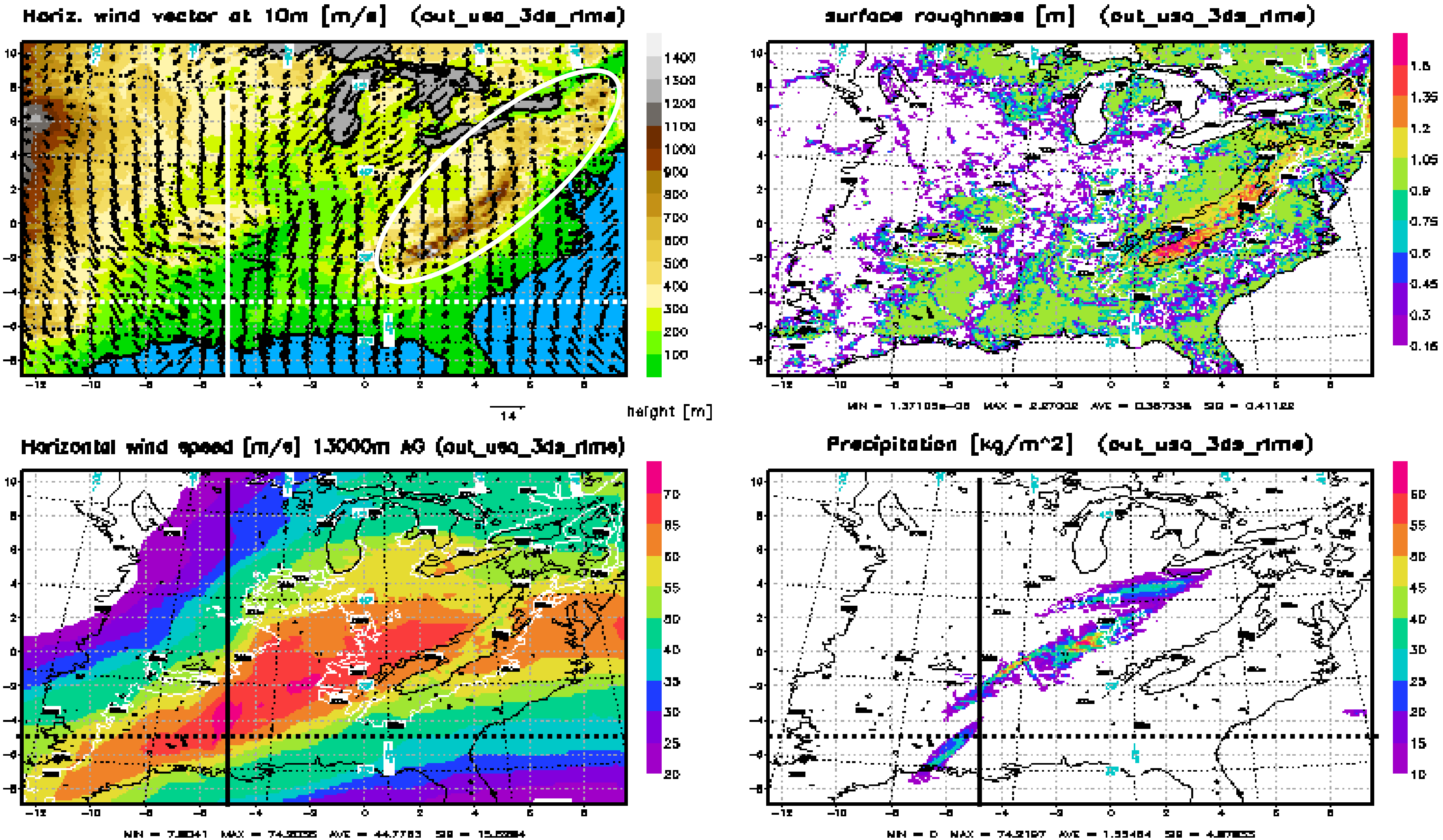
Turbulence has major impacts on aviation:

The National Center for Atmospheric Research
sponsored by NSF
NCAR
managed by UCAR to serve the community

NCAR News Release

NCAR Teams with United Airlines to Pinpoint Turbulence in Clouds;
Research Can Help Reduce Delays, Injuries, Costs
September 6, 2007

Impacts of turbulence
Turbulence has **major impacts on aviation**. According to a review of National Traffic Safety Board data from 1992 to 2001 by the National Aviation Safety Data Analysis Center, turbulence was a factor in at least **509 accidents** in the **United States**, including **251 deaths** (mostly in the general aviation sector). Additionally, the FAA Joint Safety Analysis Team estimated that there are **more than 1,000 minor turbulence-related injuries** on commercial aircraft annually. Airlines **lose millions of dollars every year due to turbulence** because of injury claims, delays, extra fuel costs, and aircraft damage.



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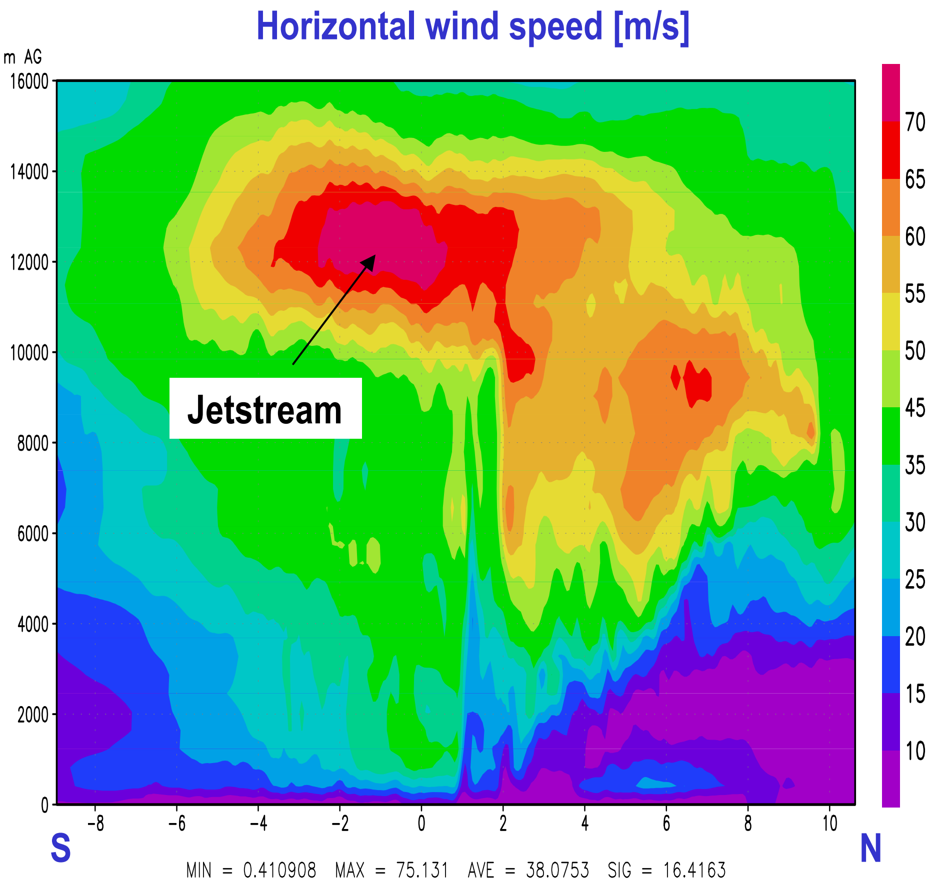
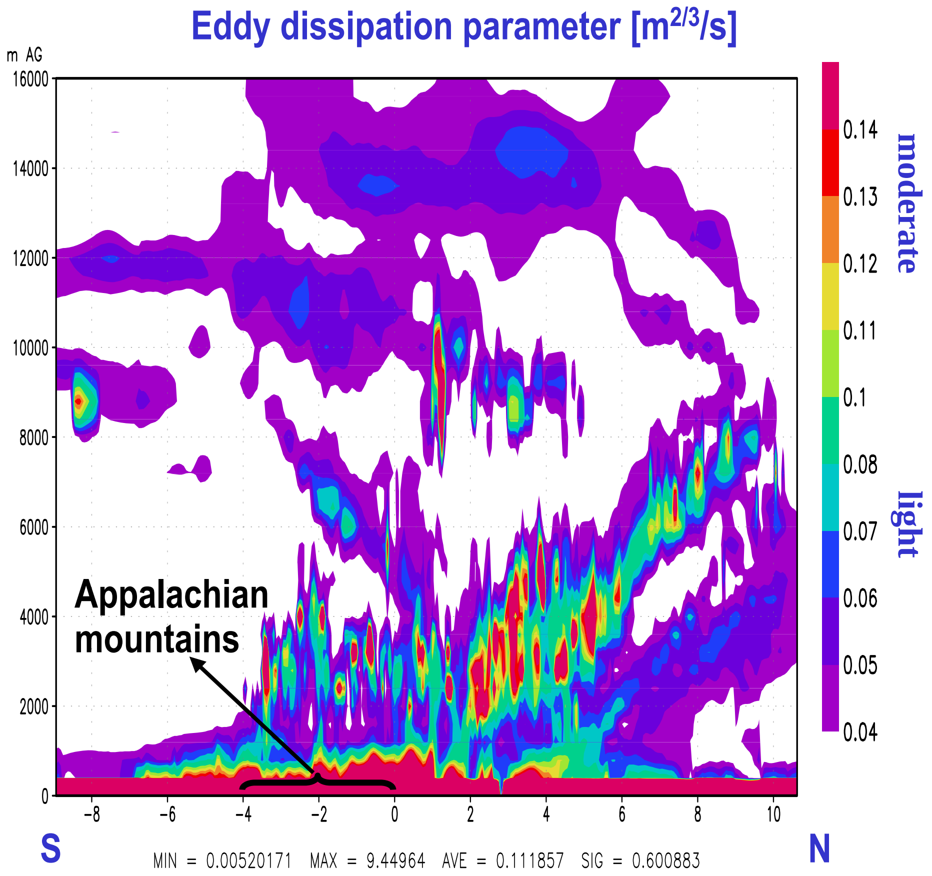
scale interaction by (positive definite **circulation term**) → **Physical based solution** also **above** the BL

Cross section of model output over the chosen USA domain

from North to South cutting the jet stream





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3D-shear production for turbulent-isotropic structures:

$$\overline{v_i' v_j'} \approx -K^M \cdot \left(\partial_j \bar{v}_i + \partial_i \bar{v}_j\right)$$

isotropic-turbulent contribution to the stress tensor

$$K^M = q \cdot S^M \cdot \ell$$

isotropic-turbulent diffusion coefficient

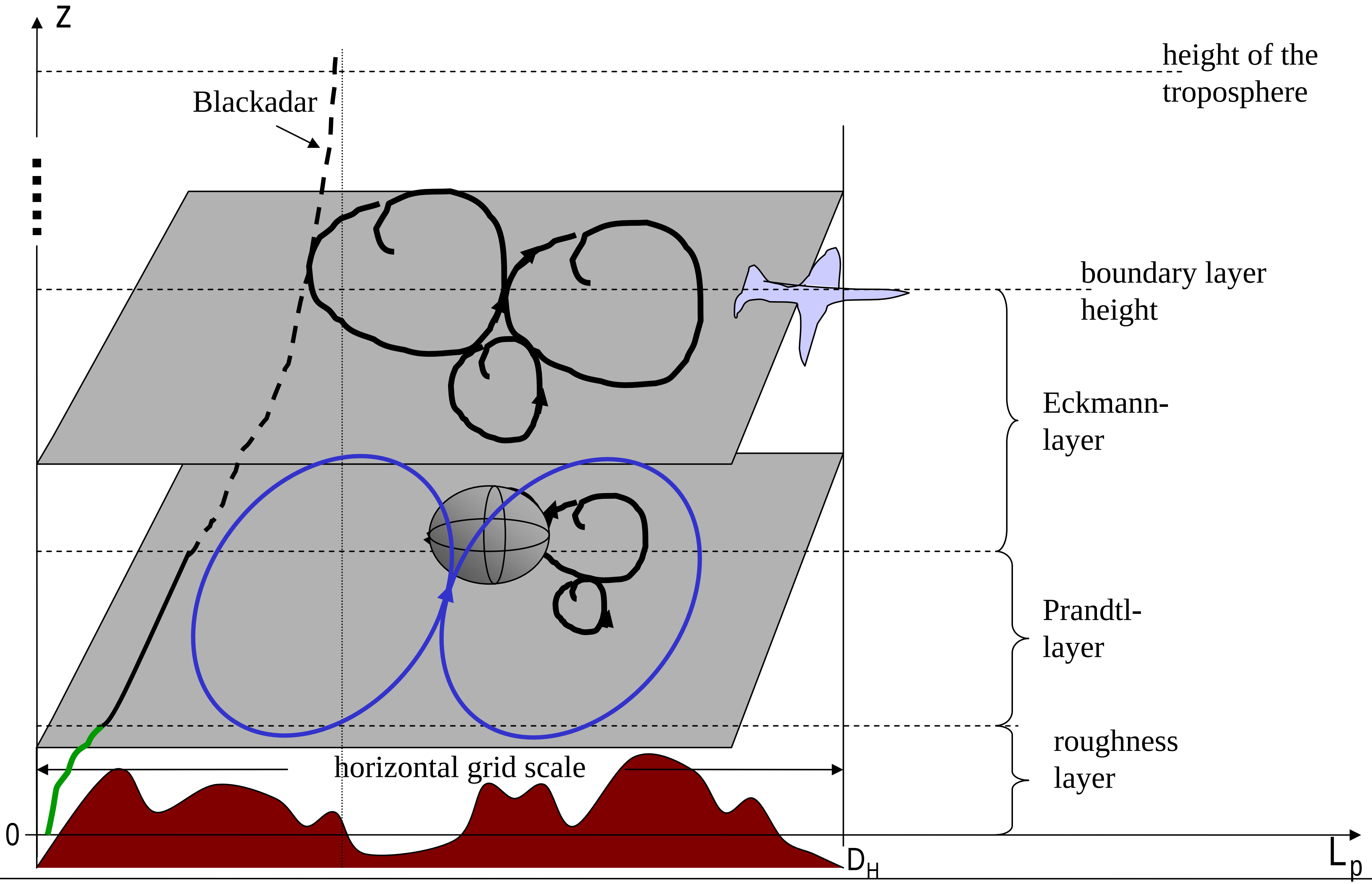
Isotropic-turbulent 3D shear term:

$$P_S := -\sum_{i=1}^{3} \overline{v_i' \underline{v'}} \cdot \nabla \bar{v}_i \approx K^M \cdot \underbrace{\sum_{i,j=1}^{3} \left(\partial_j \bar{v}_i + \partial_i \bar{v}_j\right) \cdot \partial_i \bar{v}_j}$$

$$\left(\partial_1 \bar{v}_2 + \partial_2 \bar{v}_1\right)^2 + \left(\partial_1 \bar{v}_3 + \partial_3 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_3 + \partial_3 \bar{v}_2\right)^2 + 2\left[\left(\partial_1 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_2\right)^2 + \left(\partial_3 \bar{v}_3\right)^2\right]$$

$$\xrightarrow{\text{HBA}} \left(\partial_z \bar{v}_1\right)^2 + \left(\partial_z \bar{v}_2\right)^2$$

Separated horizontal shear mode:



Parameterization of the separated horizontal shear mode:

Separated horizontal shear term:

$$P_{SH} := q_H \cdot \underbrace{\beta_H D_H}_{} \cdot \underbrace{\left[(\partial_1 \overline{v}_2 + \partial_2 \overline{v}_1)^2 + 2(\partial_1 \overline{v}_1)^2 + (\partial_2 \overline{v}_2)^2\right]}_{=: F_H^M}$$

effective mixing length of diffusion by separated horizontal shear eddies: $\beta_H < 1$ related scaling parameter

velocity scale of separated horizontal shear eddies

Equilibrium with scale transfer towards the turbulence mode:

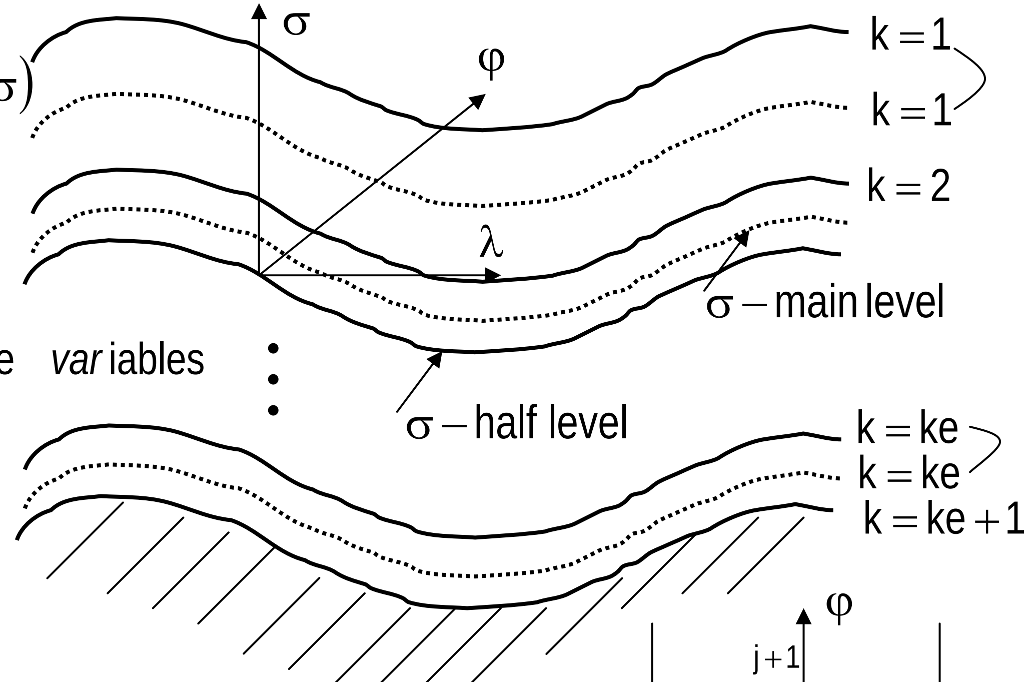
$$q_H \cdot \beta_H D_H \cdot F_H^M = \frac{q_H^3}{\alpha_H D_H}$$

$\alpha_H < 1$ scaling parameter similar to α_{EDR}

$$\Rightarrow \quad P_{SH} = q_H \beta_H D_H \cdot F_H^M = \underbrace{\alpha_H^{\frac{1}{2}} \beta_H^{\frac{3}{2}}}_{=: \alpha_S^2} D_H^2 \cdot F_H^{M\,\frac{3}{2}}$$

$=: \alpha_S^2$ effective scaling parameter

Calculation of the 3D-shear term in the COSMO-system:



COSMO-coordinates: $(\chi_1, \chi_2, \chi_3) = (\lambda, \varphi, \sigma)$

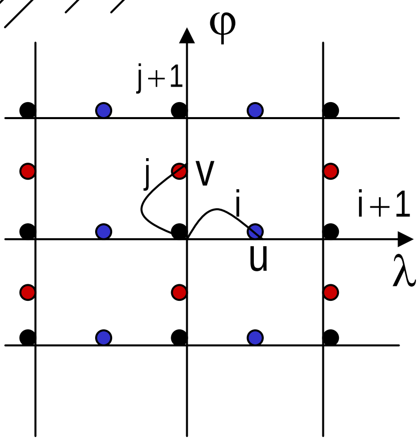
Height of a model level: $h(\lambda, \varphi, \sigma)$

Half level variables: $v_3 = w$, h, turbulence *var*iables

Main level variables:

All other scalar properties: ●

horizontal wind component: $v_1 = u$ (blue ●), $v_2 = v$ (red ●)



horizontal derivative of wind components on half levels: $n \in \{1, 2\}$; $m \in \{1, 2, 3\}$

$$\partial_n v_m = \frac{1}{h_{\chi_n}} \cdot \left(\partial_{\chi_n} - \partial_{\chi_n} h \cdot \partial_\sigma\right) v_m \approx \frac{1}{h_{\chi_n}} \cdot \left(\frac{\Delta_{\chi_{nl}} \overline{\overline{v_m}^{\chi_{p(m,n)}}}^{\sigma(m)}}{\Delta \chi_n} - \frac{\Delta_{\chi_n} h}{\Delta \chi_n} \cdot \frac{\Delta_\sigma \overline{v_m}^{\chi_{q(m)}}}{\Delta_\sigma h} \right)$$

$h_\varphi := a$ radius of the earth

Δ_χ : difference along the coordinate χ

$$p(m,l) = \begin{cases} m, & m \neq \{n,3\} \\ 0, & \text{sonst} \end{cases}$$

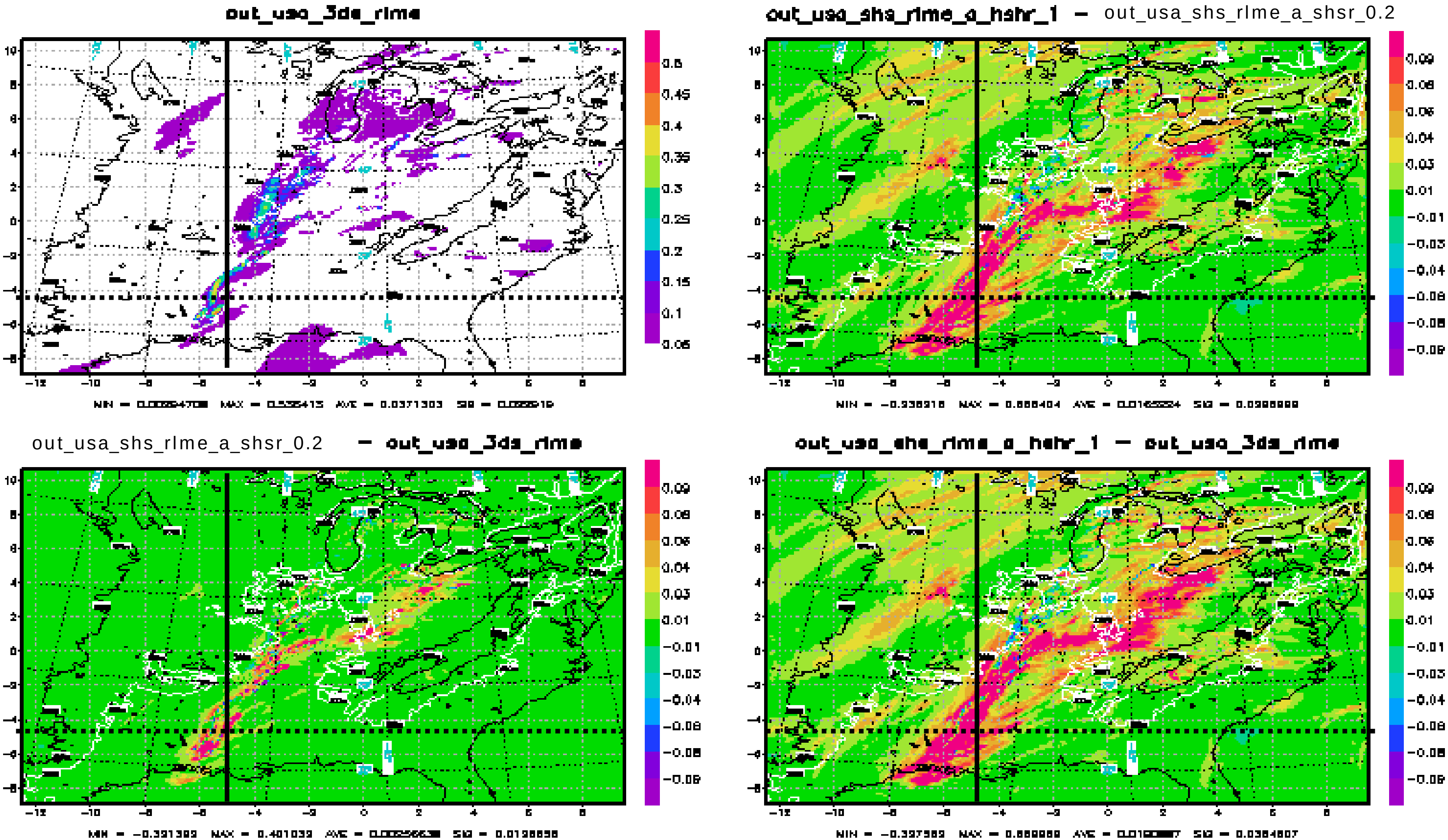
$$\sigma(m) = \begin{cases} \sigma, & m \neq 3 \\ \chi_0, & m = 3 \end{cases}$$

$\overline{\xi}^{\chi}$: interpolation along the coordinate $\chi \neq \chi_0$

$$q(m) = \begin{cases} m, & m \neq 3 \\ 0, & m = 3 \end{cases}$$

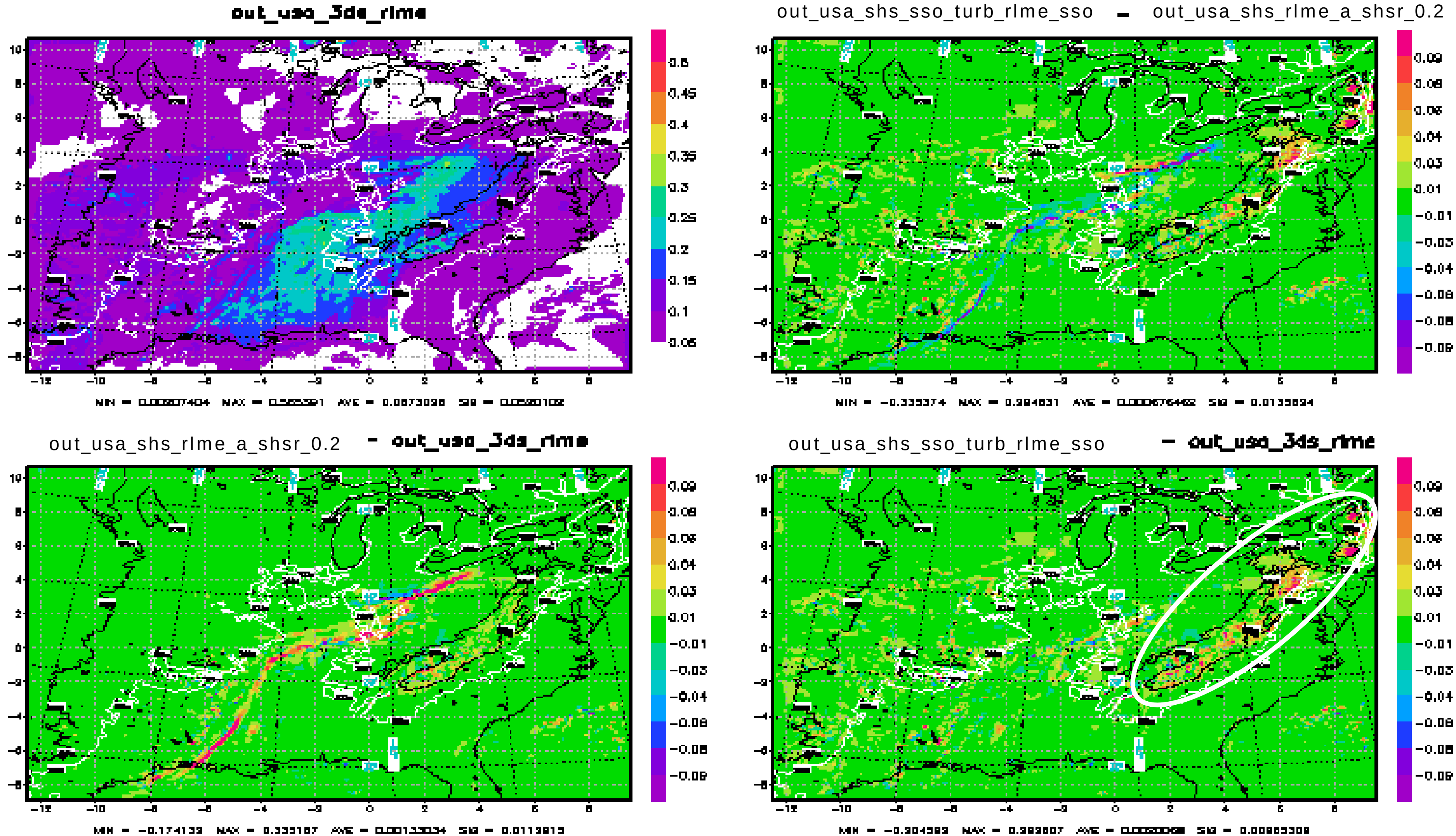
$h_\lambda := a\cos(\varphi)$

eddy dissipation parameter [m^(2/3)/s]



pr_time=06Z06FEB2008 pr_hour=6hr Lev 12 ≈ 9000m

eddy dissipation parameter [m^(2/3)/s]



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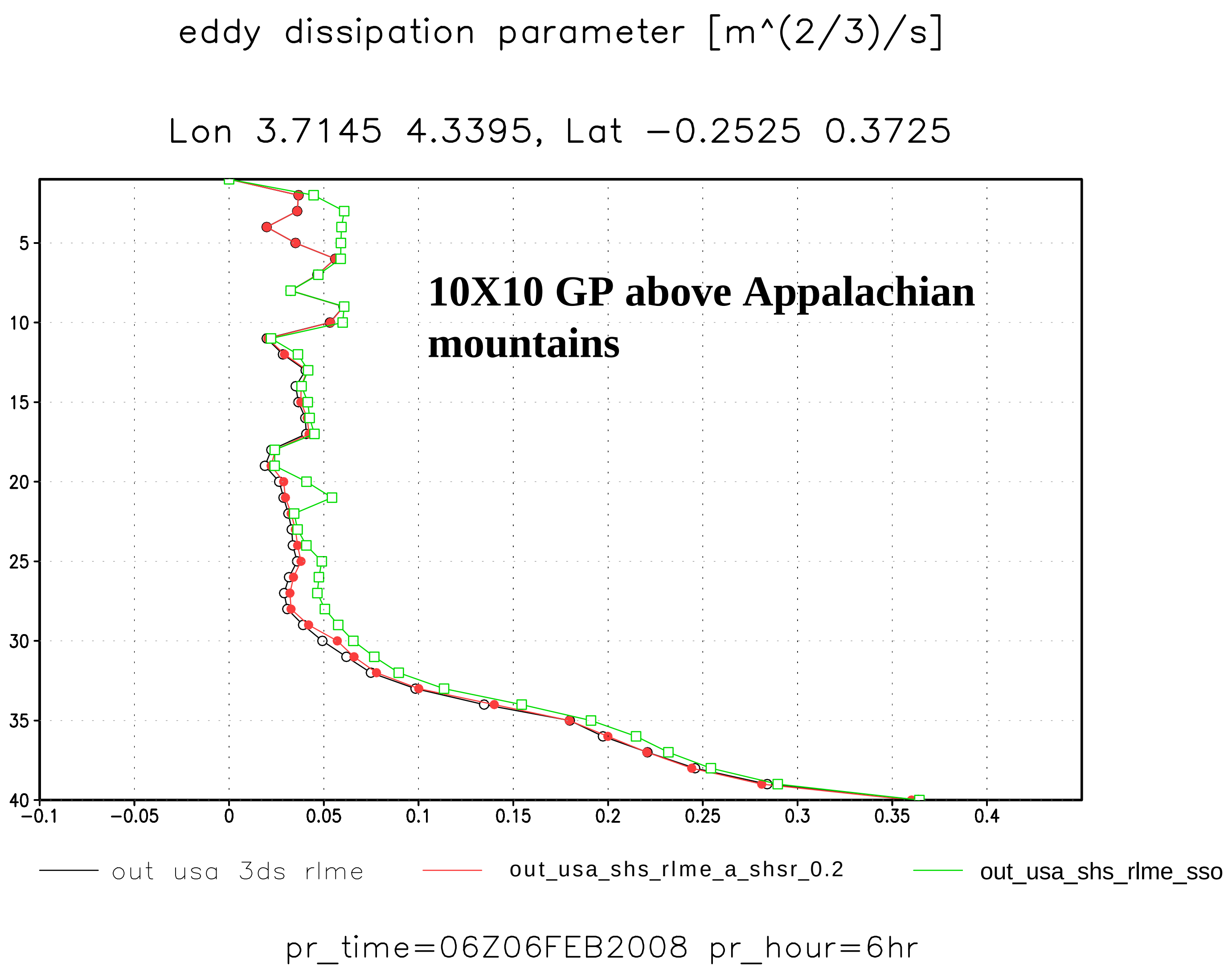


eddy dissipation parameter [m^(2/3)/s]
Lon 3.7145 4.3395, Lat −0.2525 0.3725
10X10 GP above Appalachian mountains
5
10
15
20
25
30
35
40
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
out usa 3ds rlme
out_usa_shs_rlme_a_shsr_0.2
out_usa_shs_rlme_sso
pr_time=06Z06FEB2008 pr_hour=6hr

Considered Questions:

- **What is the effect of SGS surface structures on the model equations?**
  - The influence of

    the describing coordinate system and

    the filter operator → **Roughness layer extension of the turbulence scheme**

- **How can we include these effects into a turbulence model?**
  - the topographic approximation and
  - the general boundary layer approximation

→ **Roughness layer architecture using surface area profiles**

The model equations:

- Consider the following at least 5 prognostic variables:

$\phi \in M \quad := \quad =$	$\{v_i \quad ; i=1,2,3\}$ $\{u,\ v,\ w\}$	velocity components (momentum concentration)	**momentum**
$\phi \in H \quad :=$	$\{q_w,$	mass fraction of water phases in the atmosphere	**scalars**
	$\theta_w\}$	a temperature variable related to thermal energy	

- Prognostic budget equations:

$$\partial_t(\rho\phi) + \nabla \cdot \mathbf{F}^{\phi} = Q^{\phi}$$ general budget

$F_j^{\phi} = \rho\phi v_j - c_j^{\phi}$ j-component of the total flux density for the property ϕ

ρ air density

- with the non advective molecular flux density components:

$$c_j^{\phi} = \begin{cases} -a^{\phi}\partial_j\phi & \text{for scalars} \quad \phi = \theta_w, q_w \\ -\mu\left(\partial_i v_j + \partial_j v_i\right) & \text{for momentum} \quad \phi = v_i \end{cases}$$

(i,j)-component of the reduced **molecular stress tensor**

$a^{\phi} = \rho k^{\phi}$: molecular austausch coeff. $\qquad a^{v_i} = \mu = \rho\upsilon$: isotropic dynamic viscosity

- and the source terms:

$$Q^{\phi} = \begin{cases} -\nabla\cdot \mathbf{S}/\pi c_{p_d} \approx 0 \ \text{(for moist adiab. idealization)} & ,\phi = \theta_w = \theta - \frac{L_c}{c_{p_d}} q_c \ \text{(conservative)} \\ 0 & ,\phi = q_w = q_v + q_c \ \text{conservative} \\ -\delta_{i3}\rho g - 2\rho(\boldsymbol{\Omega}\times\mathbf{v})_i - \boxed{\partial_i p} & ,\phi = v_i \ \text{not conservative} \end{cases}$$

total water potential temperature

total water mass fraction

momentum component in i-direction

$\partial_i p$ may also be regarded as a part of the i-th **diagonal** element of **molecular stress tensor**

$\mathbf{S}$: radiation flux density $\qquad \boldsymbol{\Omega}$: angular vector of the earth $\qquad \theta = T/\pi$: potential temperat.

g : gravity of the earth $\qquad \pi = \left(\frac{p}{p_r}\right)^{\frac{R_d}{c_{p_d}}}$: Exner factor $\qquad p_r = 1000$ hPa : reference pressure $\qquad p = \rho R_d\left[1+\left(\frac{R_v}{R_d}-1\right)q_v - q_c\right]T$ gas law

c_{p_d} : heat capacity of dry air at constant pressure $\qquad c_{p_v}$: heat capacity of water vapor

R_d : specific gas constant of dry air $\qquad R_v$: specific gas constant of water vapor

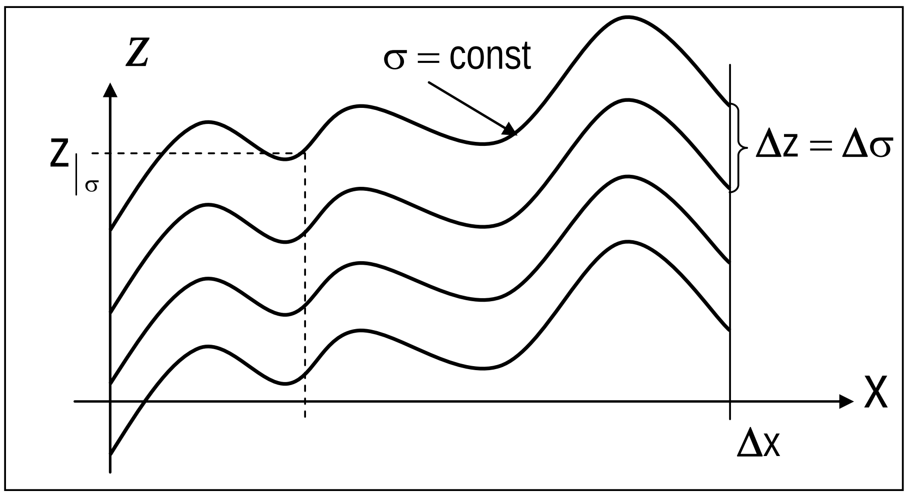
q_v : mass fraction of water vapor (specific humidity) $\qquad q_c$: mass fraction of cloud water $\qquad \begin{pmatrix} q_v \\ T \end{pmatrix} = \text{sat\_adj}\left[\begin{pmatrix} q_w \\ \theta_w \end{pmatrix}, p\right]$ saturation adjustment

The coordinate system:

- The equations must be solved on a numerical grid.
  - We choose a regular grid, belonging to a **local Cartesian** coordinate system (x, y, z) with a physical vertical coordinate σ,

    such that $z = z|_{\sigma}(x, y)$ may be **terrain following**

    and $\partial_{\sigma} z = 1$ within the boundary layer.



- The transformation T from the σ-system into the z-system with respect to a scalar field ς:

$$(x,y,\sigma)\in\Re^3 \xrightarrow{T} \left(x,y,z|_{\sigma}(x,y)\right)\in\Re^3 \xrightarrow{\varsigma|_z} \varsigma\in\Re, \qquad (x,y,\sigma)\in\Re^3 \xrightarrow{\varsigma|_{\sigma}} \varsigma\in\Re$$

- Derivatives in space transform as follows:

$$\partial_j \zeta := \partial_j \zeta|_z = \partial_j \zeta|_{\sigma} \boxed{- \partial_j z|_{\sigma}\, \partial_z \sigma \partial_{\sigma} \zeta|_{\sigma}}, \qquad j \in \{1, 2\}$$

$$\partial_3 \zeta := \partial_3 \zeta|_z = \partial_z \sigma\, \partial_{\sigma} \zeta|_{\sigma}, \qquad 3 \hat{=} z$$

The filter operator:

- In order to apply a numerical approximation of the **derivatives in space** on the variable fields, they have to be **filtered in space** with respect to a **horizontal length scale** comparable to the horizontal grid spacing $D_g = \Delta x = \Delta y$.

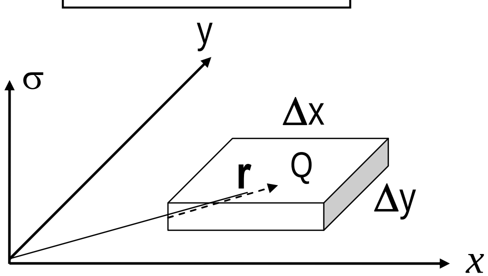
- The filter is defined as a moving average along the air containing part $G_{|\sigma}(\underline{r})$ of a shallow cubic grid box $Q_{|\sigma}(\underline{r})$ in the **σ-system** (marked with an **over-bar**):

$$\overline{\varsigma}(\mathbf{r}) := \frac{1}{\left|G_{|\sigma}(\mathbf{r})\right|} \cdot \int_{s \in G_{|\sigma}(\mathbf{r})} \varsigma_{|\sigma}(\mathbf{s})\, ds^3$$

volume average

$$\hat{\varsigma} := \frac{\overline{\rho\varsigma}}{\overline{\rho}}$$

weighted average



$$\overline{\zeta'} = 0 = \overline{\partial_j \overline{\zeta}} \qquad \overline{\rho\zeta''} = 0 = \overline{\partial_j \hat{\zeta}}$$

$$\overline{\zeta'' - \zeta'} = \overline{\zeta''} = -\frac{\overline{\rho'\zeta'}}{\overline{\rho}} = \overline{\zeta} - \hat{\zeta}$$

$$\overline{\zeta\chi} = \overline{\zeta}\,\overline{\chi} + \overline{\zeta'\chi'} \qquad \overline{\rho\zeta\chi} = \overline{\rho}\hat{\zeta}\hat{\chi} + \overline{\rho\zeta''\chi''}$$

$$\zeta'(\mathbf{s}) := \zeta(\mathbf{s}) - \overline{\zeta}(\mathbf{r})$$

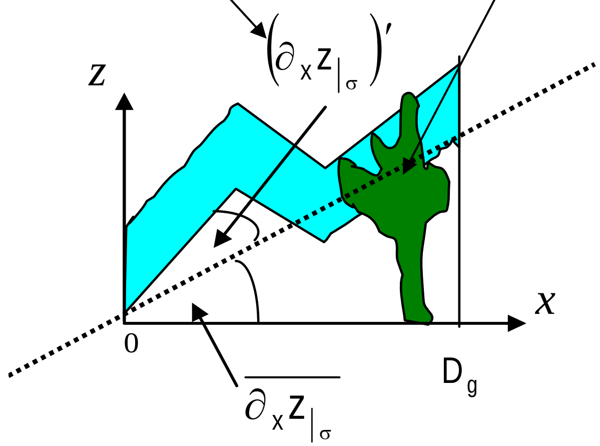
sub grid scale deviation

$$\zeta''(\mathbf{s}) := \zeta(\mathbf{s}) - \hat{\zeta}(\mathbf{r})$$

Averaged derivative along a horizontal coordinate of the Cartesian system

- **Mathematical reason for the roughness terms:**

Sub grid scale slope of the model layers or Intersections of roughness elements → Averaging and differentiation in space (flux divergence) can no longer be commuted



↓

Roughness terms in the averaged model equations

—

Wake turbulence production for TKE

The closure problem:

• Due to **nonlinearity** we get **second order statistical moments** being **additional** model variables:

e.g., we obtain for the non linear **flux term:** $\overline{\rho\phi v_j} = \overline{\rho}\hat{\phi}\hat{v}_j + \overline{\rho\phi'' v_j''}$

• **Further information** about these additional variables has to be introduced:

- we call this "**parameterization of sub grid scale processes**"
- and distinguish **different processes** more or less related to the **length scale of motions**
- each with **specific** parameterization **assumptions**

Turbulence: **isotropic**, only **<u>one</u>** characteristic **length scale** at each grid point

Circulation: **non isotropic**, **coherent** structures of **<u>additional</u> length scales,**
e.g. convection

- **1-st order closure:**

  - Parameterizations for the statistical moments are introduced **ad hoc** in the **1-st order budgets**

  - A **mass flux scheme** is a sophisticated 1-st order closure for **convection**

- **Higher order closure for turbulence:**

  - **prognostic equations** for the unknown statistical moments of order 2 (in general n) are considered

  - They contain additional statistical moments up to order n+1.

  ⟹ **Closure dilemma of hydrodynamics**

  - We use a **2-nd order closure** model according to **Mellor/Yamada** for **turbulence.**

  ⟹ **statistical moments** in the 2-nd order budgets have to be **parameterized**

General second order budget equation:

(including roughness layer terms in **topographic approximation**)

sub grid scale macroscopic transport | shear production

$$D_t\left(\overline{\rho\phi''\psi''}\right) := \partial_t\left(\overline{\rho\phi''\psi''}\right) + \overline{\nabla}\cdot\left(\overline{\rho\phi''\psi''}\,\hat{\tilde{\mathbf{v}}} + \overline{\rho\phi''\psi''\tilde{\mathbf{v}}''} + \overline{\phi''\tilde{\mathbf{e}}^{\psi}} + \overline{\psi''\tilde{\mathbf{e}}^{\phi}}\right) = -\left(\overline{\tilde{\mathbf{e}}^{\psi}}\cdot\overline{\nabla}\hat{\phi} + \overline{\tilde{\mathbf{e}}^{\phi}}\cdot\overline{\nabla}\hat{\psi}\right) - \left(\overline{\rho\psi''\tilde{\mathbf{v}}''}\cdot\overline{\nabla}\hat{\phi} + \overline{\rho\phi''\tilde{\mathbf{v}}''}\cdot\overline{\nabla}\hat{\psi}\right)$$

$$+\left(\overline{\mathbf{e}^{\psi}\cdot\nabla\phi''} + \overline{\mathbf{e}^{\phi}\cdot\nabla\psi''}\right)$$ → molecular dissipation

$$+\left(\overline{\phi'' Q^{\psi}} + \overline{\psi'' Q^{\phi}}\right)$$ → source term correlation

molecular flux density $\mathbf{e}^{\phi} := -a^{\phi}\nabla\phi$

neglected outside the **laminar** layer

vanishing for **conservative** variables

$$-\overline{\phi''\partial_i p}\quad,\quad \psi = v_i$$

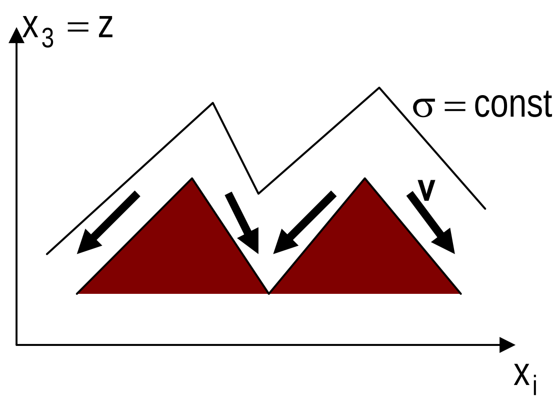
$$-\overline{\partial}_i\overline{\phi'' p'} \quad -\partial_z\left(\overline{\hat{\phi}\,p'\left(\partial_i z|_{\sigma}\right)'}\right)$$ pressure transport

$$\delta_{i3}\frac{g}{\hat{\theta}_v}\overline{\rho\phi''\theta_v''} \approx -\overline{\phi''}\,\overline{\partial}_i\overline{p} \quad + \overline{\phi'\left(\partial_i z|_{\sigma}\right)'}\;\partial_z\overline{p}$$ buoyancy source

$\approx 0,\quad \phi \notin \{v_i, v_3\}$

$$+\overline{p'\partial_i\phi''} \quad + \overline{\hat{\phi}\,\partial_z p'\left(\partial_i z|_{\sigma}\right)'}$$ pressure correlation

(return-to-isotropy) (wake production)



Postulates of pure turbulence:

- **Equilibrium** of the **source terms** in all 2-nd order budgets (if the turbulent stress tensor is substituted by its **traceless** form) :

→ - **neglect** of **local time derivative**

- **neglect** of (grid scale and sub grid scale) **transport**

- **neglect** of correlations with **source terms** of **1-st order budget equations**

- **Spectral density** of 2nd-order moments follows a **power law** in terms of wave length in **each direction** (**inertial sub range spectrum**):

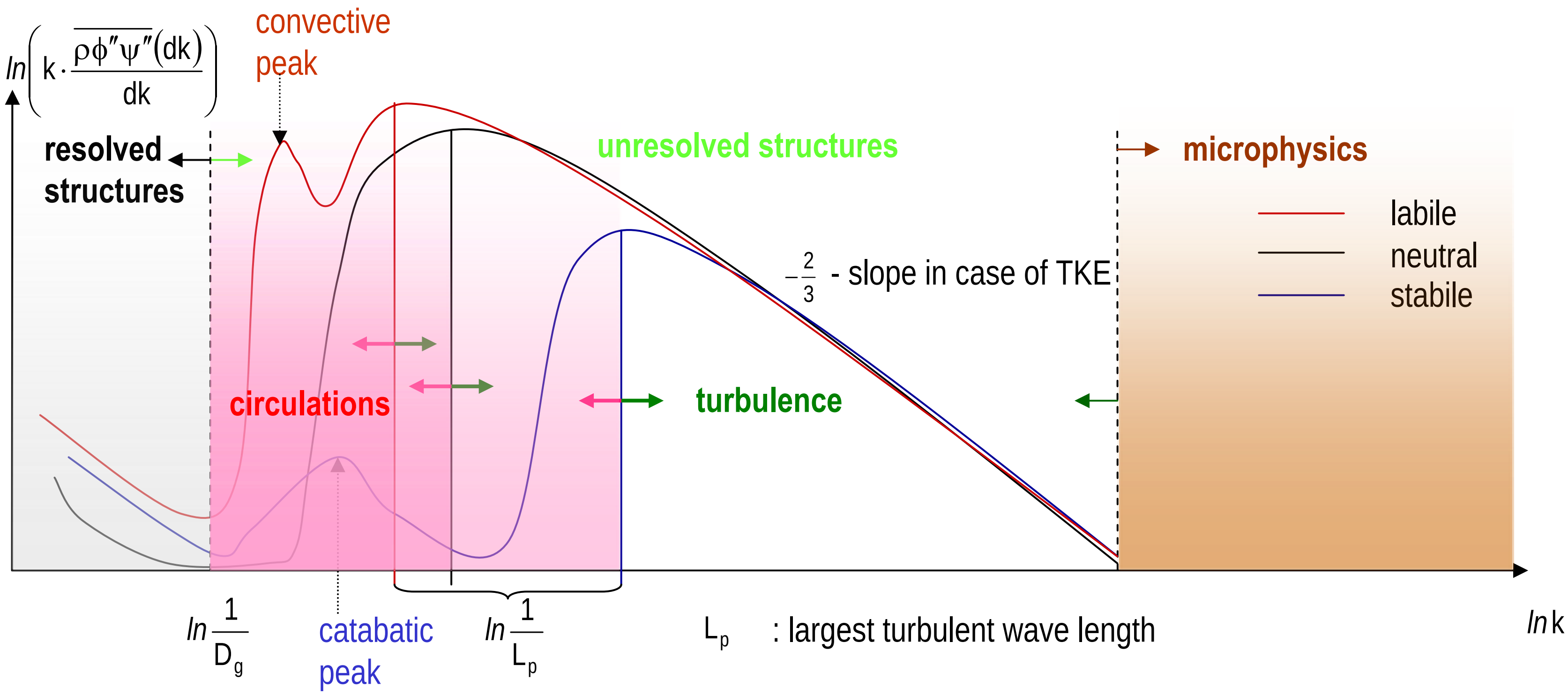
→ - whole SGS spectrum in a given sampling direction is determined by a **single** **peak wave length** L_p

- **Peak wave length** is the **same** for samples **in all directions:** **isotropic** **length scale**

→ - **pressure correlation** and **dissipation** can be closed using a **single** **turbulent master length scale** ℓ **for each location** according to **Rotta** and **Kolmogorov**

Turbulence is that class of SGS structures being in **agreement with turbulence closure assumptions!**

Spectral characteristics of turbulence and circulations:



- **circulations** generally are related with .... **additional spectral peaks**

- or they cause **different peak wavelengths** in **vertical** direction compared to the **horizontal** directions: .... **anisotropic peak wave length**

- **larger** **peak wavelength** in **vertical** direction in case of **labile** stratification
- **smaller** **peak wavelength** in **vertical** direction in case of **stabile** stratification

at least a two-scale-problem

Principle of a general valid sub grid scale parameterization:

- **Closure** of the **2-nd order budget equations** → **closure assumptions** **= further information**

- **Limited (not general valid) solution:**

  - **General valid 2-nd order closure assumptions can't exist!**
  - **Assumptions can only be valid for special conditions:** **e. g. for** sub grid scale **turbulence** ⇕ **or for** sub grid scale **convection!**

- **General valid sub grid scale closure:**

  → **Separation of sub grid scale flow in different classes**

  → **Application of specific (rather easy) closure assumptions for each class**

  → **Combination of particular parameterizations**

  } use of **different** schemes for **turbulence**, **convection** or **SSO blocking**

  → **Consideration of interaction between different classes** } **usually missing in current models!**

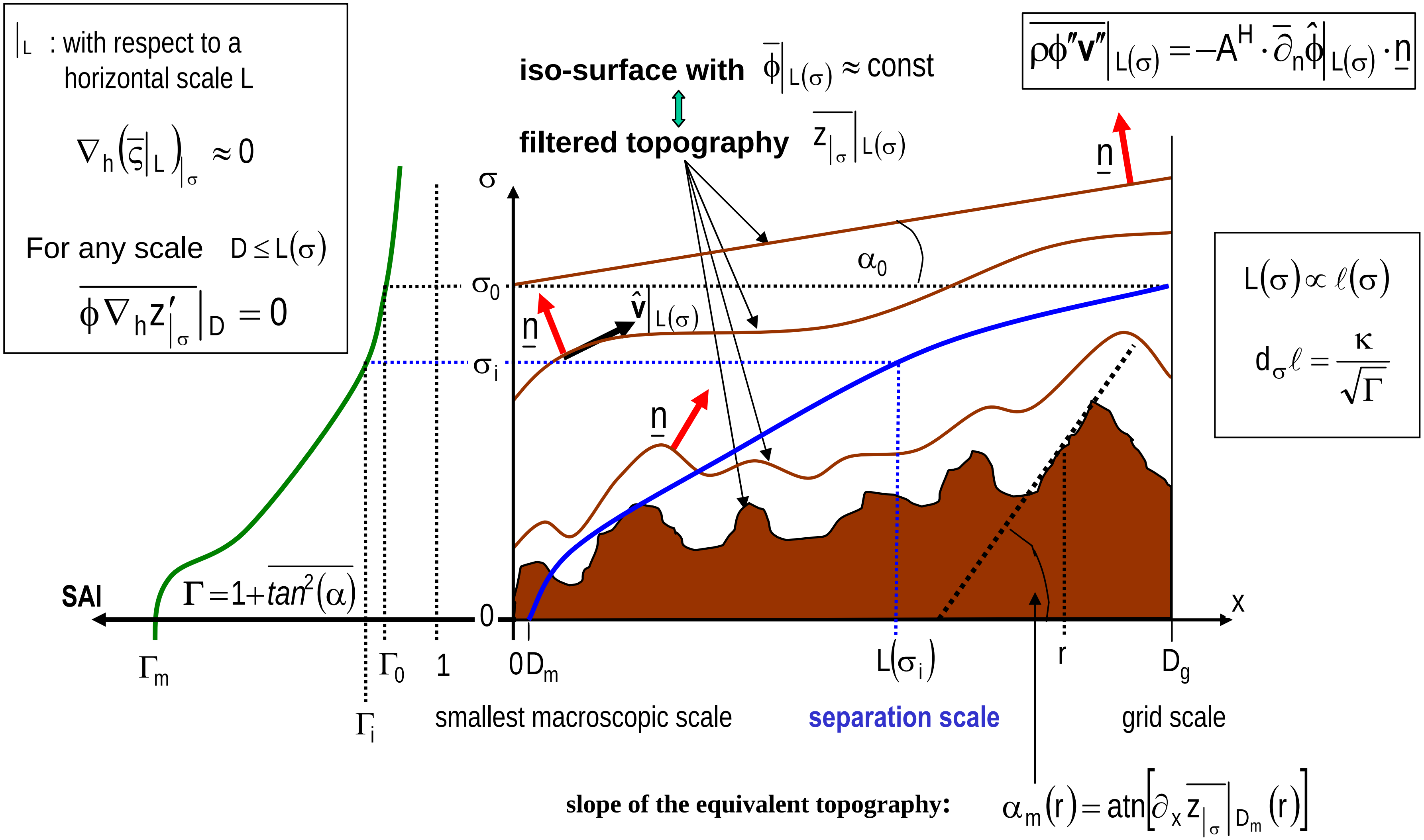
  → **Spectral separation** by

  - considering **budgets** with respect to the **separation scale** $L = min\{L_p, D_g\}$
  - **averaging** these budgets along the **whole control volume** (**double averaging**)

  → **turbulent budgets**

General boundary layer approximation GBA:

A **substitute** of the horizontal boundary layer approximation



The filtered flux divergence in GBA:

by **averaging** the **L-filtered** 1-st and 2-nd order budget equations that have been solved by applying **HBA** in an accordingly **rotated** system

$$s^2(\sigma) = \overline{\left| \nabla_h \left(\overline{z_{|\sigma}}\big|_L \right)' \right|^2}$$ **variance of the surface slope**

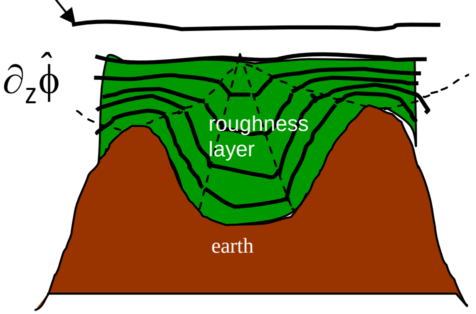
$$s_0^2 = \left| \nabla_h \bar{z}_{|\sigma} \right|^2 = \tan^2(\alpha_0)$$

$$\Gamma := 1 + s_0^2 + s^2 = 1 + \overline{\tan^2(\alpha)} = \overline{\left(\frac{1}{\cos^2 \alpha} \right)}$$ **SAI**

General boundary layer approximation:

- **iso surface** with an area being $\sqrt{\Gamma}$ times the horizontal area

$$\Gamma \cdot \overline{f_z^\phi\big|_L} = -\bar{\rho}\Gamma \underbrace{\left(1 + \frac{k^\phi}{K^\phi} \right)}_{=: r^\phi} K^\phi \partial_z \hat{\phi}$$



$$\overline{\nabla \cdot \mathbf{F}^\phi} = \overline{\nabla} \cdot \left(\bar{\rho} \hat{\phi} \hat{\mathbf{v}} \right) + \partial_z \left[\Gamma \cdot \overline{f_z^\phi\big|_L} \right]$$

effective flux divergence for **scalars**

drag flux density

$$\overline{\nabla \cdot \mathbf{F}^{v_i}} + \overline{\partial_i p} = \overline{\nabla} \cdot \left(\bar{\rho} \hat{v}_i \hat{\mathbf{v}} \right) + \bar{\partial}_i \underbrace{\left(\bar{p} + \frac{1}{3} \sum_{j=1}^{3} \overline{\rho \overline{v_j''^2}\big|_L} \right)}_{=: \rho q^2_{|L}} + \partial_z \underbrace{\left[\Gamma \cdot \overline{f_z^{v_i}} - \overline{p \left(\partial_i \overline{z_{|\sigma}}\big|_L \right)'} \right]}_{=: \left(\Gamma + \alpha^\phi s^2 \right) \cdot \overline{f_z^{v_i}}}$$

effective flux divergence for **momentum**

2 TKE

drag parameter

Stability function including circulation-, roughness- and laminar- corrections with explicit use of TKE:

$$\vartheta_v := r_T \cdot \left(r_v \vartheta_c - \frac{R_v}{R_d} \hat{T} \right) \qquad \vartheta_w := \left(\frac{R_v}{R_d} - 1 \right) \cdot \hat{\theta} + \frac{r_c}{r_p} \cdot \vartheta_v \qquad r_\theta := r_v - r_c \alpha \vartheta_v$$

$$F^M := \frac{q^2}{\ell^2} G^M = (\partial_z \hat{u})^2 + (\partial_z \hat{v})^2 \qquad G_T^M := G^M + \frac{\ell^2}{q^2} \cdot F_C^M \geq 0 \qquad F^H := \frac{q^2}{\ell^2} G^H = \frac{g}{\hat{\theta}_v} \cdot \left(\vartheta_w \partial_h \hat{q}_w + r_\theta \partial_h \hat{\theta}_w \right)$$

normal to grid scale surface

normal to horizontal surface

$$\partial_t \left(\frac{1}{2} \overline{\rho} q^2 \right) + \partial_z \left[-\overline{\rho}\, \ell S^q q \partial_z \left(\frac{1}{2} q^2 \right) \right] = \overline{\rho} \frac{q^3}{\ell} \cdot \left[r^M S^M G_T^M - S^H G^H - \frac{1}{\alpha^{MM}} \right]$$

,

$$\underbrace{\left[\frac{1}{\alpha^H} + \left(3 r^H \alpha^{HH} + 12 \alpha^M \right) \cdot G^H \right]}_{=: a_{HH}} \cdot S^H + \underbrace{6 r^M \alpha^M G_T^M}_{=: a_{HM}} \cdot S^M = 1 - 3 c^H =: b_H$$

$$\underbrace{\left[\left(9 r^H \alpha^H + 12 \alpha^M \right) \cdot G^H \right]}_{=: a_{MH}} \cdot S^H + \underbrace{\left[\frac{1}{\alpha^M} + 9 r^M \alpha^H G_T^H + 6 r^M \alpha^M G_T^M \right]}_{=: a_{MM}} \cdot S^M = 1 - 3 c^M =: b_M$$

$$\alpha^M = 0.92, \quad \alpha^H = 0.74, \quad c^M = 0.08, \quad c^H = 0.0, \quad \alpha^{MM} = 16.6, \quad \alpha^{HH} = 10.1 \qquad r^\phi := \Gamma \cdot \left(1 + \frac{k^\phi}{K^\phi} \right) \approx 1$$

$$S^H = \frac{b_H a_{MM} - b_M a_{HM}}{a_{HH} a_{MM} - a_{HM} a_{MH}} \qquad S^M = \frac{b_M a_{HH} - b_H a_{MH}}{a_{HH} a_{MM} - a_{HM} a_{MH}}$$